MHF 4U

Extra Practice – Polynomial Equations

1. Find the remainder when $3x^5 - 5x^2 + 4x + 1$ is divided by $x^2 + x - 2$.
[$r(x) = 42x - 39$]

2. If $f(x) = mx^3 + gx^2 - x + 3$ is divided by $x - 1$, the remainder is 3. If $f(x)$ is divided by $x + 3$, the remainder is -1. What are the values of m and g?
[$m = \frac{4}{9}, g = \frac{5}{9}$]

3. If $f(x) = 3x^3 + cx^2 + dx - 7$ is divided by $x - 2$, the remainder is -3. If $f(x)$ is divided by $x + 1$, the remainder is -18. What are the values of c and d?
[$c = -6$, $d = 2$]

4. If $(x - 1)$ is a factor of $x^3 - 2kx^2 + 3x + 1$, what is the value of k?
[$k = \frac{5}{2}$]

5. If $x^3 + 4x^2 + kx - 5$ is divisible by $(x + 2)$, what is the value of k?
[$k = \frac{3}{2}$]

6. Factor each sum or difference of cubes.
a) $x^3 - 27$ **b)** $y^3 + 8$ **c)** $125u^3 - 64r^3$
d) $2000w^3 + 2y^3$ **e)** $(x + y)^3 - u^3z^3$ **f)** $5u^3 - 40(x + y)^3$

7. Prove that $x^3 - 6x^2 + 3x + 10$ is divisible by $x^2 - x - 2$.

8. Prove that $(x + a)$ is a factor of $p(x) = (x + a)^5 + (x + c)^5 + (a - c)^5$.

9. Prove that $(x - a)$ is a factor of $p(x) = x^3 - (a + b + c)x^2 + (ab + bc + ca)x - abc$.

10. For each of the following, find all real and complex solutions. Provide exact solutions, in simplest radical form.
a) $(x+3)^3 - (x+1)^3 = 56$ [-5, 1]
b) $x^6 - 7x^3 - 8 = 0$ [$-1, 2, -1 \pm \sqrt{3}i, \frac{1 \pm \sqrt{3}i}{2}$]
c) $x^8 - 10x^4 + 9 = 0$ [$-1, \pm\sqrt{3}, \pm\sqrt{3}i, \pm i$