

3.1 Reciprocal of a Linear Function

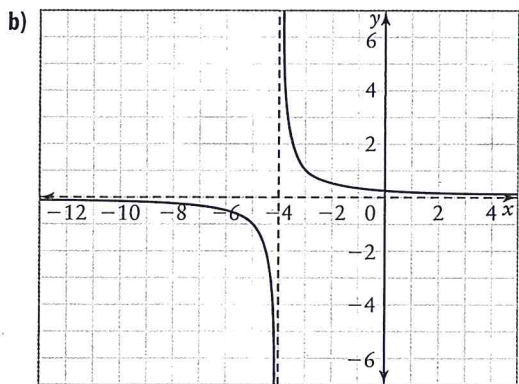
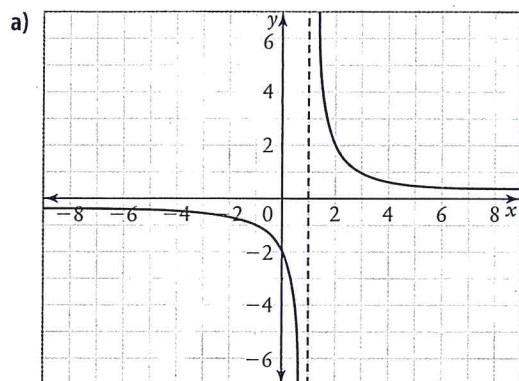
- Determine equations for the vertical and horizontal asymptotes of each function.

a) $f(x) = \frac{1}{x-2}$

b) $g(x) = \frac{3}{x+7}$

c) $h(x) = -\frac{4}{x-5}$

- Determine an equation to represent each function.



- Sketch a graph of each function. State the domain, range, y-intercepts, and equations of the asymptotes.

a) $f(x) = \frac{5}{x-3}$

b) $g(x) = -\frac{1}{x-4}$

c) $h(x) = \frac{1}{2x-3}$

d) $k(x) = -\frac{8}{5x+4}$

3.2 Reciprocal of a Quadratic Function

- Determine equations for the vertical asymptotes of each function. Then, state the domain.

a) $f(x) = \frac{1}{(x-3)(x+4)}$

b) $g(x) = -\frac{2}{(x+3)^2}$

c) $h(x) = \frac{1}{x^2+8x+12}$

- For each function,

- determine equations for the asymptotes
- determine the y-intercepts
- sketch a graph
- describe the increasing and decreasing intervals
- state the domain and range

a) $f(x) = \frac{1}{x^2+6x+5}$

b) $g(x) = \frac{1}{x^2-5x-24}$

c) $h(x) = -\frac{1}{x^2-6x+9}$

d) $k(x) = -\frac{2}{x^2+5}$

- Analyse the slope, and change in slope, for the intervals of the function $f(x) = \frac{1}{2x^2+3x-5}$ by sketching a graph of the function.

- Write an equation for a function that is the reciprocal of a quadratic and has the following properties:

- The horizontal asymptote is $y = 0$.
- The vertical asymptotes are $x = -4$ and $x = 5$.
- For the intervals $x < -4$ and $x > 5$, $y < 0$.

3.3 Rational Functions of the Form $f(x) = \frac{ax+b}{cx+d}$

- Determine an equation for the horizontal asymptote of each function.

a) $a(x) = \frac{x}{x+5}$

b) $b(x) = -\frac{2x}{x-3}$

c) $c(x) = \frac{x+2}{x-2}$

9. Summarize the key features of each function. Then, sketch a graph of the function.

a) $f(x) = \frac{x}{x-2}$ b) $g(x) = -\frac{3x}{x+1}$

c) $h(x) = \frac{x-2}{x+4}$ d) $k(x) = \frac{6x+2}{2x-1}$

10. Write an equation of a rational function of the form $f(x) = \frac{ax+b}{cx+d}$ whose graph has all of the following features:

- x-intercept of $\frac{1}{4}$
- y-intercept of $-\frac{1}{2}$
- vertical asymptote with equation $x = -\frac{2}{3}$
- horizontal asymptote with equation $y = \frac{4}{3}$

3.4 Solve Rational Equations and Inequalities

11. Solve algebraically. Check each solution.

a) $\frac{7}{x-4} = 2$ b) $\frac{3}{x^2+6x-24} = 1$

12. **Use Technology** Solve each equation using technology. If necessary, express your answers to two decimal places.

a) $\frac{4x}{x+2} = \frac{5x}{3x+1}$

b) $\frac{5x+2}{2x-9} = \frac{3x-1}{x+2}$

c) $\frac{x^2-3x+1}{2-x} = \frac{x^2+5x+4}{x-6}$

13. Solve each inequality without using technology. Illustrate the solution on a number line. Check your solutions using technology.

a) $\frac{3}{x+5} < 2$ b) $\frac{3}{x+2} \leq \frac{4}{x+3}$

c) $\frac{x^2-x-20}{x^2-4x-12} > 0$ d) $\frac{x}{x+5} > \frac{x-1}{x+7}$

14. **Use Technology** Solve each inequality using technology.

a) $\frac{x^2+5x+4}{x^2-5x+6} < 0$

b) $\frac{x^2-6x+9}{2x^2+17x+8} > 0$

3.5 Making Connections With Rational Functions and Equations

15. A manufacturer is predicting profit, P , in thousands of dollars, on the sale of x tonnes of fertilizer according to the equation $P(x) = \frac{600x - 15000}{x + 100}$.
- Sketch a graph of this relation.
 - Describe the predicted profit as sales increase.
 - Compare the rates of change of the profit at sales of 100 t and 500 t of fertilizer.
16. Sketch a graph of each function. Describe each special case.
- $f(x) = \frac{x}{x^2+5x}$
 - $g(x) = \frac{x^2-2x-35}{x^2-3x-28}$

PROBLEM WRAP-UP

The cost, C , in millions of dollars, of cleaning up an oil spill can be modelled by the function $C(p) = \frac{20}{100-p}$, where p is the percent of the oil that was spilled. The rate of change of the cost, in millions of dollars, is given by $R(p) = \frac{20}{(100-p)^2}$.

- State the domain and range of each function, and explain what they mean.
- Calculate the rate of change at $p = 50$, using the slope of the tangent to $C(p)$.
- Compare the rate of change from part b) to that using the rate of change function $R(p)$ for $p = 50$.
- Write a report outlining the cost of cleaning up the oil spill. Include the following:
 - graphs of $C(p)$ and $R(p)$
 - the total cost to clean up 25%, 50%, and 90% of the spill
 - the rate of change of the cost at 25%, 50%, and 90% of the spill