

KEY CONCEPTS

- An instantaneous rate of change corresponds to the slope of a tangent to a point on a curve.
- An approximate value for an instantaneous rate of change at a point may be determined using
 - a graph, either by estimating the slope of a secant passing through that point or by sketching the tangent and estimating the slope between the tangent point and a second point on the approximate tangent line
 - a table of values, by estimating the slope between the point and a nearby point in the table
 - an equation, by estimating the slope using a very short interval between the tangent point and a second point found using the equation

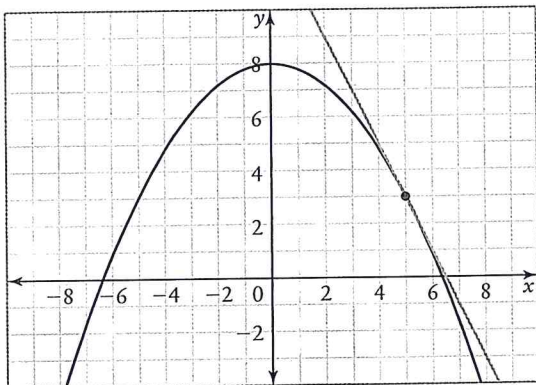
Communicate Your Understanding

- C1 a) Does the speedometer of a car measure average speed or instantaneous speed? Explain.
 - b) Describe situations in which the instantaneous speed and the average speed would be the same.
- C2 State if each situation represents average rate of change or instantaneous rate of change. Give reasons for your answer.
 - a) At 3 p.m., the plane was travelling at 850 km/h.
 - b) The average speed travelled by the train during the 10-h trip was 130 km/h.
 - c) The fire was spreading at a rate of 2 ha/h.
 - d) 5 s after an antiseptic spray is applied, the bacteria population is decreasing at a rate of 60 bacteria per second.
 - e) He lost 4 kg per month over a 5-month period.
 - f) After being heated for 2 min, the water temperature was rising at $1^{\circ}\text{C}/\text{min}$.
- C3 Which method from Examples 1, 2, and 3 is the most accurate for finding the instantaneous rate of change? Explain.

A Practise

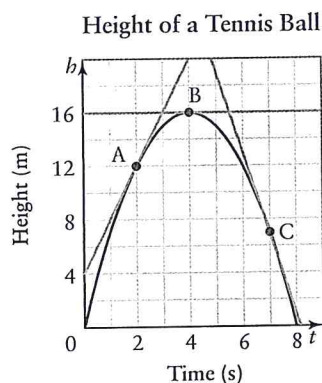
For help with question 1, refer to Example 1.

1. Consider the graph shown.



- State the coordinates of the tangent point.
- State the coordinates of another point on the tangent line.
- Use the points you found in parts a) and b) to determine the slope of the tangent line.
- What does the value you found in part c) represent?

2. a) At each of the indicated points on the graph, is the instantaneous rate of change positive, negative, or zero? Explain.



- Estimate the instantaneous rate of change at points A and C.
- Interpret the values in part b) for the situation represented by the graph.

B Connect and Apply

For help with question 3, refer to Example 3.

3. A firework is shot into the air such that its height, h , in metres, after t seconds can be modelled by the function $h(t) = -4.9t^2 + 27t + 2$.

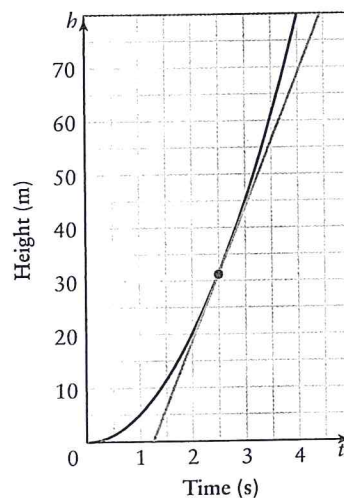
- a) Copy and complete the table.

Interval	Δh	Δt	$\frac{\Delta h}{\Delta t}$
$3 \leq t \leq 3.1$			
$3 \leq t \leq 3.01$			
$3 \leq t \leq 3.001$			

- b) Use the table to estimate the velocity of the firework after 3 s.

4. Use two different methods to estimate the slope of the tangent at the point indicated on the graph.

Distance Travelled by a Bungee Jumper



5. The data show the percent of households that play games over the Internet.

Year	% Households
1999	12.3
2000	18.2
2001	24.4
2002	25.7
2003	27.9

Source: Statistics Canada, Canada at a Glance 2006, page 9, Household Internet use at home by Internet activity.

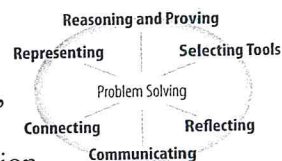
- Determine the average rate of change, in percent, of households that played games over the Internet from 1999 to 2003.
 - Estimate the instantaneous rate of change in percent of households that played games over the Internet
 - in 2000
 - in 2002
 - Compare the values found in parts a) and b). Explain any similarities and differences.
6. The table shows the consumer price index (CPI) every 5 years from 1955 to 2005.

Year	CPI
1955	16.8
1960	18.5
1965	20.0
1970	24.2
1975	34.5
1980	52.4
1985	75.0
1990	93.3
1995	104.2
2000	113.5
2005	127.3

Source: Statistics Canada, CANSIM Table 326-0002.

- Determine the average rate of change in the CPI from 1955 to 2005.
- Estimate the instantaneous rate of change in the CPI for
 - 1965
 - 1985
 - 2000
- Compare the values found in parts a) and b). Explain any similarities and differences.

7. A soccer ball is kicked into the air such that its height, h , in metres, after t seconds can be modelled by the function $h(t) = -4.9t^2 + 12t + 0.5$.



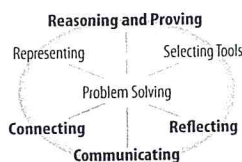
- Determine the average rate of change of the height of the ball from 1 s to 2 s.
 - Estimate the instantaneous rate of change of the height of the ball after 1 s.
 - Sketch the curve and the tangent.
 - Interpret the average rate of change and the instantaneous rate of change for this situation.
8. On Earth, the height, h , in metres, of a free-falling object after t seconds can be modelled by the function $h(t) = -4.9t^2 + k$, while on Venus, the height can be modelled by $h(t) = -4.45t^2 + k$, where $t \geq 0$ and k is the height, in metres, from which the object is dropped. Suppose a rock is dropped from a height of 60 m on each planet.
- Determine the average rate of change of the height of the rock in the first 3 s after it is dropped.
 - Estimate the instantaneous rate of change of the height of the rock 3 s after it is dropped.
 - Interpret the values in parts a) and b) for this situation.
9. A manufacturer estimates that the cost, C , in dollars, of producing x MP3 players can be modelled by $C(x) = 0.000\ 15x^3 + 100x$.
- Determine the average rate of change of the cost of producing from 100 to 200 MP3 players.
 - Estimate the instantaneous rate of change of the cost of producing 200 MP3 players.
 - Interpret the values found in parts a) and b) for this situation.
 - Does the cost ever decrease? Explain.

CONNECTIONS

The CPI measures the average price of consumer goods and services purchased by households. The percent change in the CPI is one measure of inflation.

10. Suppose the revenue, R , in dollars, from the sales of x MP3 players described in question 9 is given by

$$R(x) = x(350 - 0.000\,325x^2).$$



- Determine the average rate of change of revenue from selling from 100 to 200 MP3 players.
- Estimate the instantaneous rate of change of revenue from the sale of 200 MP3 players.
- Interpret the values found in parts a) and b) for this situation.
- The profit, P , in dollars, from the sale of x MP3 players is given by the profit function $P(x) = R(x) - C(x)$. Determine an equation for the profit function.
- Determine the average rate of change of profit on the sale of from 100 to 200 MP3 players.
- Estimate the instantaneous rate of change of profit on the sale of 200 MP3 players.
- Interpret the values found in parts e) and f) for this situation.

11. A worldwide distributor of basketballs determines that the yearly profit, P , in thousands of dollars, earned on the sale of x thousand basketballs can be modelled by the function $P(x) = -0.09x^3 + 1.89x^2 + 9x$, where $x \in [0, 25]$.

- Determine the average rate of change of profit earned on the sale of from
 - 2000 to 6000 basketballs
 - 16 000 to 20 000 basketballs
- What conclusions can you make by comparing the values in part b)? Explain your reasoning.
- Estimate the instantaneous rate of change of profit earned on the sale of
 - 5000 basketballs
 - 18 000 basketballs
- What conclusions can you make from the values found in parts c)? Explain your reasoning.
- Use Technology** Graph the function. How does the graph support your answers in parts a) and c)?

C Extend and Challenge

12. The population, P , of a small town after t years can be modelled by the function $P(t) = 0.5t^3 + 150t + 1200$, where $t = 0$ represents the beginning of this year.
- Write an expression for the average rate of change of the population from $t = 8$ to $t = 8 + h$.
 - Use the expression in part a) to determine the average rate of change of the population when
 - $h = 2$
 - $h = 4$
 - $h = 5$
 - What do the values you found in part b) represent?
 - Describe how the expression in part a) could be used to estimate the instantaneous rate of change of the population after 8 years.
 - Use the method you described in part d) to estimate the instantaneous rate of change of the population after 8 years.
13. **Math Contest** Determine the exact value of $\sqrt{10 + \sqrt{10 + \sqrt{10 + \dots}}}$.
14. **Math Contest** Find $|m|$ given $\sqrt[3]{m + 9} = 3 + \sqrt[3]{m - 9}$.