EX 2.2 (p. 101 – 103) # 1 – 15, AND #18

The zeros are 1, -2, and $\frac{1}{3}$. The corresponding factors are $x - 1$, $x + 2$, and $3x - 1$.

So, $3x^3 + 2x^2 - 7x + 2 = (x - 1)(x + 2)(3x - 1)$.

Possible dimensions of the rectangular block of ice, in metres, are $x - 1$, $x + 2$, and $3x - 1$.

b) For $x = 1.5$,

$x - 1 = 1.5 - 1$
$= 0.5$

$x + 2 = 1.5 + 2$
$= 3.5$

$3x - 1 = 3(1.5) - 1$
$= 4.5 - 1$
$= 3.5$

When $x = 1.5$, the dimensions are 0.5 m by 3.5 m by 3.5 m.

In Example 4, once one factor is determined for a polynomial whose leading coefficient is not 1, you can use division to determine the other factors.

KEY CONCEPTS

For integer values of a and b with $a \neq 0$,

- The factor theorem states that $x - b$ is a factor of a polynomial $P(x)$ if and only if $P(b) = 0$.
  Similarly, $ax - b$ is a factor of $P(x)$ if and only if $P\left(\frac{b}{a}\right) = 0$.
- The integral zero theorem states that if $x - b$ is a factor of a polynomial function $P(x)$ with leading coefficient 1 and remaining coefficients that are integers, then b is a factor of the constant term of $P(x)$.
- The rational zero theorem states that if $P(x)$ is a polynomial function with integer coefficients and $x = \frac{b}{a}$ is a rational zero of $P(x)$, then
  - b is a factor of the constant term of $P(x)$
  - a is a factor of the leading coefficient of $P(x)$
  - $ax - b$ is a factor of $P(x)$

Communicate Your Understanding

C1 **a)** Which of the following binomials are factors of the polynomial $P(x) = 2x^3 + x^2 - 7x - 6$? Justify your answers.

A $x - 1$ **B** $x + 1$ **C** $x + 2$ **D** $x - 2$ **E** $2x + 1$ **F** $2x + 3$

b) Use the results of part a) to write $P(x) = 2x^3 + x^2 - 7x - 6$ in factored form.

C2 When factoring a trinomial $ax^2 + bx + c$, you consider the product ac. How does this relate to the rational zero theorem?

C3 Describe the steps required to factor the polynomial $2x^3 - 3x^2 + 5x - 4$.

C4 Identify the possible factors of the expression $x^3 + 2x^2 - 5x - 4$. Explain your reasoning.

A Practise

For help with questions 1 and 2, refer to Example 1.

1. Write the binomial factor that corresponds to the polynomial $P(x)$.

a) $P(4) = 0$ **b)** $P(-3) = 0$

c) $P\left(\frac{2}{3}\right) = 0$ **d)** $P\left(-\frac{1}{4}\right) = 0$

2. Determine if $x + 3$ is a factor of each polynomial.

a) $x^3 + x^2 - x + 6$

b) $2x^3 + 9x^2 + 10x + 3$

c) $x^3 + 27$

For help with question 3, refer to Example 2.

3. List the values that could be zeros of each polynomial. Then, factor the polynomial.

a) $x^3 + 3x^2 - 6x - 8$

b) $x^3 + 4x^2 - 15x - 18$

c) $x^3 - 3x^2 - 10x + 24$

For help with question 4, refer to Example 3.

4. Factor each polynomial by grouping terms.

a) $x^3 + x^2 - 9x - 9$

b) $x^3 - x^2 - 16x + 16$

c) $2x^3 - x^2 - 72x + 36$

d) $x^3 - 7x^2 - 4x + 28$

e) $3x^3 + 2x^2 - 75x - 50$

f) $2x^4 + 3x^3 - 32x^2 - 48x$

For help with question 5, refer to Example 4.

5. Determine the values that could be zeros of each polynomial. Then, factor the polynomial.

a) $3x^3 + x^2 - 22x - 24$

b) $2x^3 - 9x^2 + 10x - 3$

c) $6x^3 - 11x^2 - 26x + 15$

d) $4x^3 + 3x^2 - 4x - 3$

B Connect and Apply

6. Factor each polynomial.

a) $x^3 + 2x^2 - x - 2$

b) $x^3 + 4x^2 - 7x - 10$

c) $x^3 - 5x^2 - 4x + 20$

d) $x^3 + 5x^2 + 3x - 4$

e) $x^3 - 4x^2 - 11x + 30$

f) $x^4 - 4x^3 - x^2 + 16x - 12$

g) $x^4 - 2x^3 - 13x^2 + 14x + 24$

7. **Use Technology** Factor each polynomial.

a) $8x^3 + 4x^2 - 2x - 1$

b) $2x^3 + 5x^2 - x - 6$

c) $5x^3 + 3x^2 - 12x + 4$

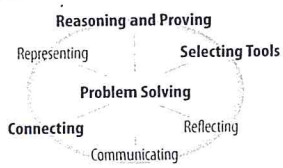
d) $6x^4 + x^3 - 8x^2 - x + 2$

e) $5x^4 + x^3 - 22x^2 - 4x + 8$

f) $3x^3 + 4x^2 - 35x - 12$

g) $6x^3 - 17x^2 + 11x - 2$

8. An artist creates a carving from a rectangular block of soapstone whose volume, V, in cubic metres, can be modelled by $V(x) = 6x^3 + 25x^2 + 2x - 8$. Determine possible dimensions of the block, in metres, in terms of binomials of x.



9. Determine the value of k so that $x + 2$ is a factor of $x^3 - 2kx^2 + 6x - 4$.

10. Determine the value of k so that $3x - 2$ is a factor of $3x^3 - 5x^2 + kx + 2$.

11. Factor each polynomial.

a) $2x^3 + 5x^2 - x - 6$

b) $4x^3 - 7x - 3$

c) $6x^3 + 5x^2 - 21x + 10$

d) $4x^3 - 8x^2 + 3x - 6$

e) $2x^3 + x^2 + x - 1$

f) $x^4 - 15x^2 - 10x + 24$

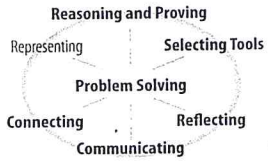
12. a) Factor each difference of cubes.

i) $x^3 - 1$

ii) $x^3 - 8$

iii) $x^3 - 27$

iv) $x^3 - 64$



b) Use the results of part a) to predict a pattern for factoring $x^3 - a^3$.

c) Use your pattern from part b) to factor $x^3 - 125$. Verify your answer by expanding.

d) Factor each polynomial.

i) $8x^3 - 1$ ii) $125x^6 - 8$

iii) $64x^{12} - 27$ iv) $\frac{8}{125}x^3 - 64y^6$

13. a) Factor each sum of cubes.

i) $x^3 + 1$ ii) $x^3 + 8$

iii) $x^3 + 27$ iv) $x^3 + 64$

b) Use the results of part a) to predict a pattern for factoring $x^3 + a^3$.

c) Use your pattern from part b) to factor $x^3 + 125$. Verify your answer by expanding.

d) Factor each polynomial.

i) $8x^3 + 1$ ii) $125x^6 + 8$

iii) $64x^{12} + 27$ iv) $\frac{8}{125}x^3 + 64y^6$

14. Show that $x^4 + x^2 + 1$ is non-factorable over the integers.

15. Factor by letting $m = x^2$.

a) $4x^4 - 37x^2 + 9$

b) $9x^4 - 148x^2 + 64$

✓Achievement Check

16. Chapter Problem Best of U has produced a new body wash. The profit, P, in dollars, can be modelled by the function $P(x) = x^3 - 6x^2 + 9x$, where x is the number of bottles sold, in thousands.

a) Use the factor theorem to determine if $x - 1$ is a factor of $P(x)$.

b) Use the rational zero theorem to write the possible values of $\frac{b}{a}$ for the factored form: $P(x) = x(x^2 - 6x + 9)$

c) Use long division to check that $x - 3$ is a factor.

d) The company is happy with the profit and manufactured a similar body spray. The profit of this product can be modelled by the function $P(x) = 4x^3 + 12x^2 - 16x$. Find the factors of $P(x)$.

C Extend and Challenge

17. Factor each polynomial.

a) $2x^5 + 3x^4 - 10x^3 - 15x^2 + 8x + 12$

b) $4x^6 + 12x^5 - 9x^4 - 51x^3 - 30x^2 + 12x + 8$

18. Determine the values of m and n so that the polynomials $2x^3 + mx^2 + nx - 3$ and $x^3 - 3mx^2 + 2nx + 4$ are both divisible by $x - 2$.

19. Determine a polynomial function $P(x)$ that satisfies each set of conditions.

a) $P(-4) = P\left(-\frac{3}{4}\right) = P\left(\frac{1}{2}\right) = 0$ and $P(-2) = 50$

b) $P(3) = P(-1) = P\left(\frac{2}{3}\right) = P\left(-\frac{3}{2}\right) = 0$ and $P(1) = -18$

20. a) Factor each expression.

i) $x^4 - 1$ ii) $x^4 - 16$

iii) $x^5 - 1$ iv) $x^5 - 32$

b) Use the results of part a) to predict a pattern for factoring $x^n - a^n$.

c) Use your pattern from part b) to factor $x^6 - 1$. Verify your answer by expanding.

d) Factor each expression.

i) $x^4 - 625$ ii) $x^5 - 243$

21. Is there a pattern for factoring $x^n + a^n$? Justify your answer.

22. Math Contest When a polynomial is divided by $(x + 2)$, the remainder is -19. When the same polynomial is divided by $(x - 1)$, the remainder is 2. Determine the remainder when the polynomial is divided by $(x - 1)(x + 2)$.