

# EX 3.1 (p.153-155) #1-13

## Communicate Your Understanding

- C1 The calculator screen gives a table of values for the function  $f(x) = \frac{4}{x-3}$ . Explain why there is an error statement.
- C2 a) For the reciprocal of any linear function, as the denominator increases, what happens to the function value?  
b) For the reciprocal of any linear function, as the denominator approaches zero, what happens to the function value?
- C3 Can you find an example of a linear function whose reciprocal has no restrictions on either the domain or range? If yes, give an example. If no, explain.

X	Y1
2.999	-400
2.999	-4000
2.9999	-40000
3.0001	40000
3.001	4000
3.01	400
X=2.99	

## A Practise

For questions 1 and 2, refer to Example 1.

1. Copy and complete each table to describe the behaviour of the function as  $x$  approaches each key value.

a)  $f(x) = \frac{1}{x-2}$

As $x \rightarrow$	$f(x) \rightarrow$
$2^+$	
$2^-$	
$+\infty$	
$-\infty$	

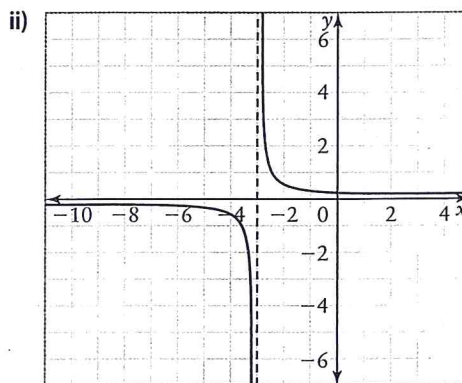
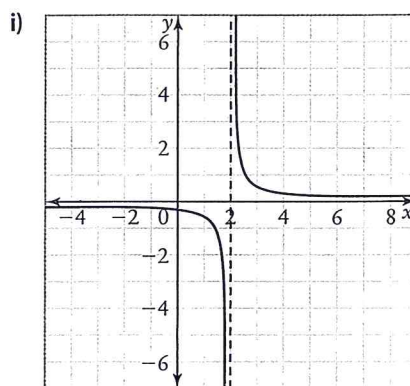
b)  $f(x) = \frac{1}{x+5}$

As $x \rightarrow$	$f(x) \rightarrow$
$-5^+$	
$-5^-$	
$+\infty$	
$-\infty$	

c)  $f(x) = \frac{1}{x+8}$

As $x \rightarrow$	$f(x) \rightarrow$
$-8^+$	
$-8^-$	
$+\infty$	
$-\infty$	

2. a) Write equations to represent the horizontal and vertical asymptotes of each rational function.



- b) Write a possible equation for each function in part a).

For help with questions 3 to 5, refer to Example 2.

3. For each reciprocal function,
- write an equation to represent the vertical asymptote
  - write an equation to represent the horizontal asymptote
  - determine the y-intercept

a)  $f(x) = \frac{1}{x-5}$

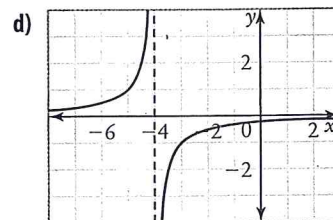
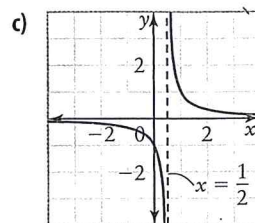
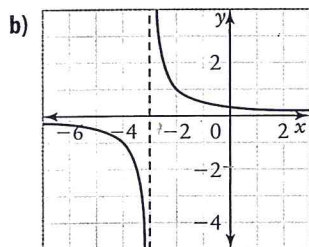
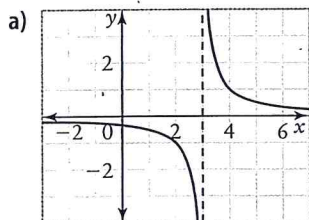
b)  $g(x) = \frac{2}{x+6}$

c)  $h(x) = \frac{5}{1-x}$

d)  $k(x) = -\frac{1}{x+7}$

4. **Use Technology** Verify the vertical asymptotes in question 3 using technology.

5. Determine a possible equation to represent each function shown.



For help with question 6, refer to Example 3.

6. Sketch each function and then describe the intervals where the slope is increasing and the intervals where it is decreasing.

a)  $f(x) = \frac{1}{x-3}$

b)  $k(x) = \frac{3}{2x+7}$

c)  $m(x) = -\frac{2}{x+4}$

d)  $p(x) = \frac{5}{3-2x}$

## B Connect and Apply

7. Sketch a graph of each function. Label the y-intercept. State the domain, range, and equations of the asymptotes.

a)  $f(x) = \frac{1}{x-1}$

b)  $g(x) = \frac{1}{x+4}$

c)  $h(x) = \frac{1}{2x+1}$

d)  $k(x) = -\frac{1}{x+4}$

e)  $m(x) = -\frac{3}{2x-5}$

f)  $n(x) = \frac{4}{5-x}$

g)  $p(x) = \frac{1}{\left(x - \frac{1}{4}\right)}$

h)  $q(x) = -\frac{3}{\left(x + \frac{1}{2}\right)}$

8. Determine the equation in the form

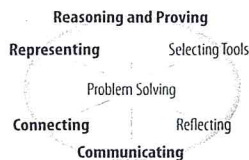
$f(x) = \frac{1}{kx-c}$  for the function with a vertical asymptote at  $x = 1$  and a y-intercept at  $-1$ .

9. Determine the equation in the form

$f(x) = \frac{1}{kx-c}$  for the function with a vertical asymptote at  $x = -1$  and a y-intercept at  $-0.25$ .



10. The time required to fly from one location to another is inversely proportional to the average speed. When the average speed to fly from Québec City to Vancouver is 350 km/h, the flying time is 11 h.



- Write a function to represent the time as a function of the speed.
- Sketch a graph of this function.
- How long would the trip from Québec to Vancouver take at an average speed of 500 km/h?
- Describe the rate of change of the time as the average speed increases.

#### CONNECTIONS

If two variables,  $x$  and  $y$ , are inversely proportional, then  $y = \frac{k}{x}$ , where  $k$  is a constant.

- Investigate a variety of functions of the form  $f(x) = \frac{1}{bx + 2}$ .
- What is the effect on the graph when the value of  $b$  is varied?

12. Use the results from question 11 to sketch a graph of each function.

a)  $f(x) = \frac{1}{x - 5}$

b)  $g(x) = \frac{1}{2x - 5}$

c)  $h(x) = \frac{1}{3x - 5}$

13. The force required to lift an object is inversely proportional to the distance of the force from the fulcrum of a lever. A force of 200 N is required at a point 3 m from the fulcrum to lift a certain object.

- Determine a function to represent the force as a function of the distance.
- Sketch a graph of this function.
- How much force is required to lift this object at a point 2 m from the fulcrum?
- What is the effect on the force needed as the distance from the fulcrum is doubled?

#### CONNECTIONS

A fulcrum is the pivot on which a lever turns.

### C Extend and Challenge

14. Analyse the key features (domain, range, vertical asymptotes, and horizontal asymptotes) of each function, and then sketch the function.

a)  $f(x) = \frac{1}{\sqrt{x}}$

b)  $g(x) = \frac{1}{|x|}$

c)  $f(x) = \frac{3}{x - 2} + 4$

15. Graph the line  $y = 2x - 5$  and find the  $x$ -intercept. Analyse the reciprocals of the  $y$ -coordinates on either side of the  $x$ -intercept. How do these numbers relate to the key features of the function  $f(x) = \frac{1}{2x - 5}$ ?

16. Math Contest Solve for  $x$  in terms of  $y$  and  $z$ :

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$$

17. Math Contest Given that  $3a = 75b$ , find the value of  $\frac{3a - 5b}{5b}$ .

18. Math Contest Two points are chosen on the unit circle with centre at the origin. What is the probability that the distance between these two points is at least 1?

A  $\frac{1}{4}$     B  $\frac{1}{2}$     C  $\frac{3}{4}$     D  $\frac{1}{3}$     E  $\frac{2}{3}$