Ex #3.3 (p. 174–175) #1–10, 13.

A Practise

For help with questions 1 to 3, refer to Example 1.

1. Determine an equation for the vertical asymptote of each function. Then, state the domain.

a) $f(x) = \dfrac{x}{x - 7}$ b) $g(x) = \dfrac{2x}{x + 5}$

c) $h(x) = -\dfrac{x}{x + 8}$ d) $k(x) = \dfrac{x}{3x - 1}$

e) $m(x) = \dfrac{5x - 3}{4x + 9}$ f) $n(x) = \dfrac{6 - x}{5 - x}$

2. Determine an equation for the horizontal asymptote of each function. Then, state the range.

a) $p(x) = \dfrac{x}{x - 6}$ b) $q(x) = \dfrac{3x}{x + 4}$

c) $r(x) = \dfrac{x - 1}{x + 1}$ d) $s(x) = \dfrac{5x - 2}{2x + 3}$

e) $t(x) = \dfrac{x - 6}{4 - x}$ f) $u(x) = \dfrac{3 - 4x}{1 - 2x}$

3. Sketch each function and then summarize the increasing and decreasing intervals.

a) $f(x) = \dfrac{x}{x - 5}$ b) $c(x) = \dfrac{4x}{x + 8}$

c) $k(x) = \dfrac{x + 1}{4 - x}$ d) $w(x) = \dfrac{x + 2}{4x - 5}$

e) $d(x) = \dfrac{-2x - 3}{x + 5}$ f) $m(x) = \dfrac{3x + 1}{2x + 1}$

For help with questions 4 to 6, refer to Example 2.

4. a) For the function $f(x) = \dfrac{2x}{x - 3}$, compare the slopes of the tangents

i) at the points where $x = 3.5$ and $x = 20$

ii) at the points where $x = 2.5$ and $x = -20$

b) What do these results indicate about the key features of the graph?

B Connect and Apply

5. a) Determine an equation for the horizontal asymptote of each function.

i) $f(x) = \dfrac{x - 5}{2x + 1}$ ii) $g(x) = \dfrac{3 - 5x}{2x + 1}$

b) How do the equations of the horizontal asymptotes relate to the coefficients in each function?

c) Summarize your findings to describe how to determine an equation for the horizontal asymptote of the function $f(x) = \dfrac{ax + b}{cx + d}$.

6. Use your results from question 5 to determine an equation for the horizontal asymptote of each function. Then, determine an equation for the vertical asymptote and graph the function. State the domain and range.

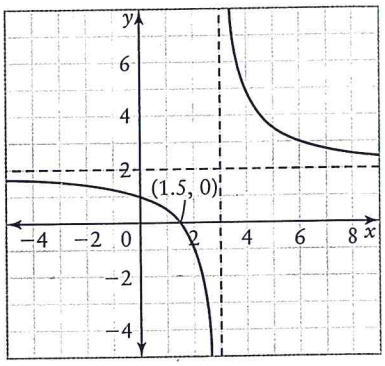
a) $f(x) = \dfrac{x}{x - 9}$ b) $g(x) = \dfrac{3x}{x + 2}$

c) $h(x) = \dfrac{4x - 3}{2x + 1}$ d) $k(x) = \dfrac{x - 3}{2x - 5}$

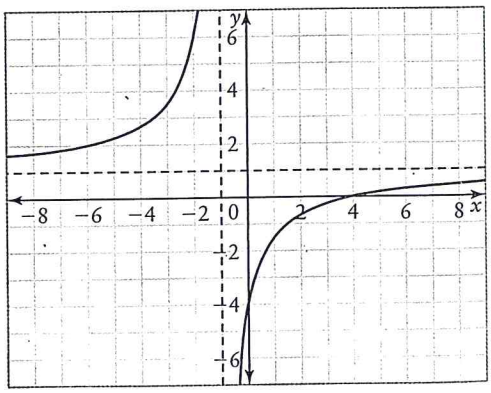
e) $m(x) = \dfrac{4 - x}{5 + x}$ f) $p(x) = \dfrac{6 - 8x}{3x - 4}$

7. Write an equation for the rational function shown on each graph.

a)



b)



8. Write an equation for a rational function whose graph has all of the indicated features.
   - x-intercept of -4
   - y-intercept of -2
   - vertical asymptote with equation $x = 2$
   - horizontal asymptote with equation $y = 1$

9. Write an equation for a rational function whose graph has all of the indicated features.
   - x-intercept of $\frac{3}{5}$
   - y-intercept of -3
   - vertical asymptote with equation $x = -\frac{1}{2}$
   - horizontal asymptote with equation $y = \frac{5}{2}$

10. **Chapter Problem** After a train derailment in Northern Ontario, the concentration, C, in grams per litre, of a pollutant after t minutes in a 5 000 000-L pond can be modelled by the function $C(t) = \frac{30t}{200\,000 + t}$, when a pollutant concentration of 30 g/L flows into the pond at a rate of 25 L/min.
    a) Sketch a graph showing the concentration of the pollutant after t minutes.
    b) What happens as t becomes very large?
    c) When a concentration level of 0.05 g/L in the pond is reached, the fish stock will be irreversibly damaged. When will this occur?

11. a) Use long division to rewrite the function $f(x) = \frac{4x + 5}{2x - 1}$ as the sum of a constant and a rational function.
    b) Explain how this method could be used to graph rational functions.
    c) Use this method to sketch a graph of $f(x)$.

12. Use your method from question 11 to graph each function.
    a) $p(x) = \frac{2x + 3}{x + 1}$
    b) $t(x) = \frac{5x - 4}{2x + 5}$

✓Achievement Check

13. Consider the function $g(x) = \frac{x}{x - 7}$.
    a) Determine an equation for the vertical asymptote.
    b) State the domain.
    c) Determine an equation for the horizontal asymptote.
    d) State the range.
    e) Sketch the function.
    f) Summarize the increasing and decreasing intervals.
    g) Compare the slopes of the tangents at the points where
       i) $x = 7.5$ and $x = 20$
       ii) $x = 6.5$ and $x = -20$

C Extend and Challenge

14. A golf ball of mass 4.6 g is struck by a golf club at a speed of 50 m/s. The ball has initial velocity, v, in metres per second, of $v(m) = \frac{83m}{m + 0.046}$, where m is the mass, in grams, of the golf club. Describe the rate of change of the initial velocity as the mass of the club increases.

15. Analyse the key features of the function $f(x) = \frac{\sqrt{x}}{\sqrt{x} - 1}$ and sketch its graph. How does it compare to the graph of $f(x) = \frac{x}{x - 1}$?