Ex 1.5 (p. 189) #1–10
(–p 191)

KEY CONCEPTS

- When solving a problem, it is important to read carefully to determine whether a function is being analysed or an equation or inequality is to be solved.
- A full analysis will involve four components:
  - numeric (tables, ordered pairs, calculations)
  - algebraic (formulas, solving equations)
  - graphical
  - verbal (descriptions)
- When investigating special cases of functions, factor and reduce where possible. Indicate the restrictions on the variables in order to identify hidden discontinuities.
- When investigating new types of rational functions, consider what is different about the coefficients and the degree of the polynomials in the numerator and denominator. These differences could affect the stretch factor of the curve and the equations of the asymptotes and they could cause other discontinuities.

Communicate Your Understanding

C1 Other than at asymptotes, describe when a discontinuity can occur in a rational function.

C2 The maximum height, h, in kilometres, of a rocket launched with initial velocity, v, in kilometres per hour, is given by the formula $h = \frac{6400v^2}{125\,440 - v^2}$, ignoring air resistance. Explain why the velocity at the vertical asymptote is considered the escape velocity of the rocket.

CONNECTIONS

Escape velocity is the speed at which the magnitude of the kinetic energy of an object is equal to the magnitude of its gravitational potential energy. It is commonly described as the speed needed to "break free" from a gravitational field.

A Practise

For help with question 1, refer to Example 1.

1. The intensity of illumination is inversely proportional to the square of the distance to the light source. It is modelled by the formula $I = \frac{k}{d^2}$, where I is the intensity, in lux; d is the distance, in metres, from the source; and k is a constant. When the distance from a certain light source is 50 m, the intensity is 6 lux.

a) Sketch a graph of this relation.

b) Describe what happens to the light intensity as the distance becomes greater.

c) Comment on the model for values of d close to 0.

For help with questions 2 and 3, refer to Example 2.

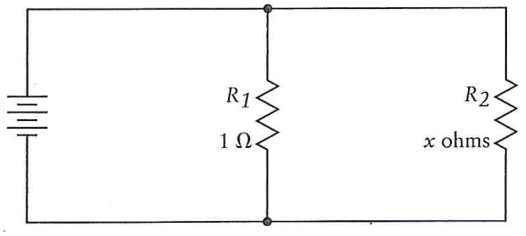
2. According to Boyle's law, under constant temperature, the volume of gas varies inversely with the pressure. A tank holds 10 L of hydrogen gas at a pressure of 500 kPa.

a) Determine a function to relate volume and pressure for this gas.

b) Sketch a graph of this relation, showing the volume of gas for different atmospheric pressures.

c) What is the effect of doubling the pressure?

3. When connected in parallel, a resistor of x ohms and a resistor of 1 Ω will have a total resistance defined by the function $R(x) = \frac{x}{1 + x}$.



a) For the total resistance to be 0.5 Ω, what does the resistance x need to be?

b) For the total resistance to be less than 0.25 Ω, what does the resistance x need to be?

For help with question 4, refer to Example 3.

4. Sketch a graph of each function. Describe each special case.

a) $f(x) = \frac{x}{x^2 - 2x}$

b) $g(x) = \frac{x - 4}{x^2 + x - 20}$

c) $h(x) = \frac{x^2 + 7x + 12}{x^2 + 2x - 3}$

d) $k(x) = \frac{3x^2 + x - 2}{2x^2 + 7x + 5}$

e) $m(x) = \frac{x}{x^3 - 4x^2 - 12x}$

f) $n(x) = \frac{x^2 - 3x + 2}{x^3 - 7x + 6}$

B Connect and Apply

5. The profit, P, in thousands of dollars, from the sale of x kilograms of coffee can be modelled by the function $P(x) = \frac{4x - 200}{x + 400}$.

a) Sketch a graph of this relation.

b) The average profit for a given sales level, x, can be found by drawing a secant from the origin to the point $(x, P(x))$. Explain how average profit is related to the slope of a secant.

c) Estimate where the average profit is the greatest. Verify using slopes.

d) Determine the rate of change of the profit at a sales level of 1000 kg.

6. The electrical resistance, R, in ohms, of a wire varies directly with its length, l, in metres, and inversely with the square of the diameter, d, in millimetres, of its cross section according to the function $R = \frac{kl}{d^2}$.

a) If 1000 m of 4-mm-diameter wire has a resistance of 40 Ω, determine an equation to model length and cross section.

b) Sketch a graph of the function for the electrical resistance of 1000 m of wire at various cross sections.

7. A bus company models its cost, C, in dollars, per person for a bus charter trip with the equation $C(x) = \frac{10\,000}{10 + x}$, where x is the number of passengers over its minimum number of 10. Describe the change in the cost model represented by each of the following, and accompany each with a graph.

a) $C(x) = \frac{10\,000}{8 + x}$

b) $C(x) = \frac{20\,000}{10 + x}$

c) $C(x) = \frac{15\,000}{12 + x}$

8. A function has equation $f(x) = \frac{x^2 - 2x - 5}{x - 1}$. In addition to a vertical asymptote, it has an oblique, or slant, asymptote, which is neither vertical nor horizontal.

a) **Use Technology** Graph the function using technology.

b) Describe what is meant by the term *oblique asymptote*.

c) Use long division to rewrite this function.

d) How can the new form of the equation be used to determine an equation for the slant asymptote?

9. Refer to question 8 to graph each function.

a) $f(x) = \dfrac{x^2 + 5x - 2}{x + 3}$

b) $g(x) = \dfrac{2x^2 - 5x + 3}{x + 2}$

10. In the event of a power failure, a computer model estimates the temperature, T, in degrees Celsius, in a food-processing plant's freezer to be $T = \dfrac{2t^2}{t + 1} - 15$, where t is the time, in hours, after the power failure.

a) Sketch a graph of this function. Use technology or the method from question 8.

b) How long would it take for the temperature to reach 0°C?

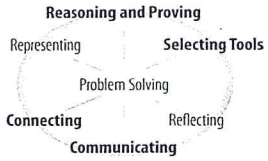
c) A generator starts up when the temperature is −5°C. How long would it take for this to happen?

11. A cylindrical tank is to have a volume of 100 000 cm^3.

a) Write a formula for the height in terms of the radius.

b) Sketch a graph of the relationship between height and radius.

12. **Use Technology** As blood moves away from the heart, the systolic pressure, P, in millimetres of mercury (mmHg), after t seconds, changes according to the function $P(t) = \dfrac{25t^2 + 125}{t^2 + 1}$, $0 \leq t \leq 10$.



a) Graph this function using technology.

b) Describe what happens to the systolic pressure over the first 10 s.

c) To measure the rate of change in systolic pressure, you can use the function $R(t) = -\dfrac{200t}{(t^2 + 1)^2}$. Graph this function using technology. Describe the rate of change.

d) Compare the rate of change at $t = 5$ s, using the slope of a tangent of $P(t)$, to the rate of change function $R(t)$ at $t = 5$ s.

C Extend and Challenge

13. For $x > 0$, what value of x gives the least sum of x and its reciprocal?

14. The function $C(t) = \dfrac{0.16t}{t^2 + t + 2}$ models the concentration, C, in milligrams per cubic centimetre, of a drug in the bloodstream after time, t, in minutes.

a) Sketch a graph of the function, without using technology.

b) Explain the shape of the graph in the context of the concentration of the drug in the bloodstream.

15. A generator produces electrical power, P, in watts, according to the function $P(R) = \dfrac{120R}{(0.4 + R)^2}$, where R is the resistance, in ohms. Determine the intervals on which the power is increasing.

16. **Math Contest** Is the following statement true or false? The function $g(x) = \dfrac{x^n - n^2}{x^{n-1} - n}$, $n \in \mathbb{N}$, has a slant asymptote for all values of n. Give a reason for your answer.

17. **Math Contest** When the polynomial $5x^4 + 4x^3 + 3x^2 + Px + Q$ is divided by $x^2 - 1$, the remainder is 0. What is the value of $P + Q$?

A −12 **B** 212 **C** 26 **D** 12 **E** 6

18. **Math Contest** Solve the equation $\sqrt{3}\sin x + \cos x = 2$ for x.