

2.1 The Remainder Theorem

1. i) Use the remainder theorem to determine the remainder for each division.
 ii) Perform each division. Express the result in quotient form. Identify any restrictions on the variable.
 - a) $x^3 + 9x^2 - 5x + 3$ divided by $x - 2$
 - b) $12x^3 - 2x^2 + x - 11$ divided by $3x + 1$
 - c) $-8x^4 - 4x + 10x^3 - x^2 + 15$ divided by $2x - 1$
2. a) Determine the value of k such that when $f(x) = x^4 + kx^3 - 3x - 5$ is divided by $x - 3$, the remainder is -10 .
 b) Determine the remainder when $f(x)$ is divided by $x + 3$.
 c) **Use Technology** Verify your answer in part b) using technology.
3. For what value of b will the polynomial $P(x) = 4x^3 - 3x^2 + bx + 6$ have the same remainder when it is divided by $x - 1$ and by $x + 3$?

2.2 The Factor Theorem

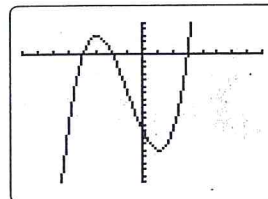
4. Factor each polynomial.
 - a) $x^3 - 4x^2 + x + 6$
 - b) $3x^3 - 5x^2 - 26x - 8$
 - c) $5x^4 + 12x^3 - 101x^2 + 48x + 36$
5. Factor.
 - a) $-4x^3 - 4x^2 + 16x + 16$
 - b) $25x^3 - 50x^2 - 9x + 18$
 - c) $2x^4 + 5x^3 - 8x^2 - 20x$
6. Rectangular blocks of limestone are to be cut up and used to build the front entrance of a new hotel. The volume, V , in cubic metres, of each block can be modelled by the function $V(x) = 2x^3 + 7x^2 + 2x - 3$.
 - a) Determine the dimensions of the blocks in terms of x .
 - b) What are the possible dimensions of the blocks when $x = 1$?

7. Determine the value of k so that $x + 3$ is a factor of $x^3 + 4x^2 - 2kx + 3$.

2.3 Polynomial Equations

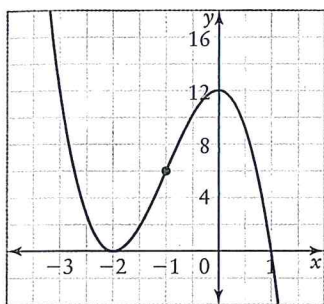
8. Use the graph to determine the roots of the corresponding polynomial equation.

Window variables: $x \in [-8, 8]$,
 $y \in [-40, 10]$, $Y_{\text{scl}} = 2$



9. Determine the real roots of each equation.
 - a) $(5x^2 + 20)(3x^2 - 48) = 0$
 - b) $(2x^2 - x - 13)(x^2 + 1) = 0$
 10. Solve. Round answers to one decimal place, if necessary.
 - a) $7x^3 + 5x^2 - 5x - 3 = 0$
 - b) $-x^3 + 9x^2 = x + 6$
 11. The specifications for a cardboard box state that the width is 5 cm less than the length, and the height is 1 cm more than double the length. Write an equation for the volume of the box and find possible dimensions for a volume of 550 cm^3 .
- ## 2.4 Families of Polynomial Functions
12. Examine the following functions. Which function does not belong to the same family? Explain.
 - A $y = 3.5(x + 2)(x - 1)(x - 3)$
 - B $y = -0.2(x - 3)(2x + 4)(2x - 3)$
 - C $y = (4x - 12)(x + 2)(x - 1)$
 - D $y = -7(x - 1)(x - 3)(x + 2)$
 13. a) Determine an equation, in simplified form, for the family of cubic functions with zeros $2 \pm \sqrt{5}$ and 0.
 b) Determine an equation for the member of the family with graph passing through the point $(2, 20)$.

14. Determine an equation for the function represented by this graph.



2.5 Solving Inequalities Using Technology

15. **Use Technology** Solve. Round the zeros to one decimal place, if necessary.

- $x^2 + 3x - 5 \geq 0$
- $2x^3 - 13x^2 + 17x + 12 > 0$
- $x^3 - 2x^2 - 5x + 2 < 0$
- $3x^3 + 4x^2 - 35x - 12 \leq 0$
- $-x^4 - 2x^3 + 4x^2 + 10x + 5 < 0$

16. **Use Technology** A section of a water tube ride at an amusement park can be modelled by the function $h(t) = -0.002t^4 + 0.104t^3 - 1.69t^2 + 8.5t + 9$, where t is the time, in seconds, and h is the height, in metres, above the ground. When will the riders be more than 15 m above the ground?

2.6 Solving Factorable Polynomial Inequalities Algebraically

17. Solve each inequality. Show the solution on a number line.

- $(5x + 4)(x - 4) < 0$
- $-(2x + 3)(x - 1)(3x - 2) \leq 0$
- $(x^2 + 4x + 4)(x^2 - 25) > 0$

18. Solve by factoring.

- $12x^2 + 25x - 7 \geq 0$
- $6x^3 + 13x^2 - 41x + 12 \leq 0$
- $-3x^4 + 10x^3 + 20x^2 - 40x + 32 < 0$

PROBLEM WRAP-UP

Best of U has developed a new cologne and perfume and is now searching for artistic designs for the crystal bottles that will contain these products. The company has decided to run a promotional design contest on its Web site. Here are the specifications. Your task is to prepare an entry for the contest.

Best of U
Crystal Bottle Design Contest

The Design Component

Submit the designs.

- Create a pair of similar bottle designs, one for the cologne and one for the perfume.
- Draw each design as a two-dimensional representation on a grid with a scale.
- Create each design using the graphs of a family (or families) of polynomial functions.
- The design may be drawn by hand or with the use of technology.

The Written Component

Submit a written component.

- Write equation(s) for the family (or families) of polynomial functions and the members of each family used in your designs.
- Solve two equations that correspond to the functions in the design whose graphs cross the x-axis. State the real roots and the x-intercepts.
- Write two inequalities (one using \geq and one using \leq) using two different functions in the design. Solve each inequality.