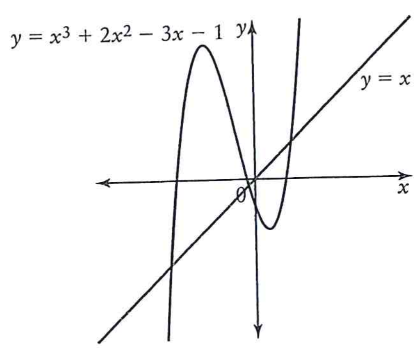
KEY CONCEPTS

Key Features of Graphs of Polynomial Functions With Odd Degree

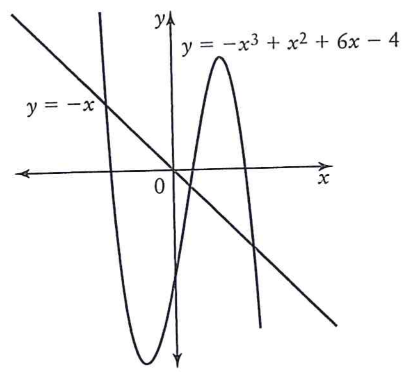
Positive Leading Coefficient

- the graph extends from quadrant 3 to quadrant 1 (similar to the graph of $y = x$)



Negative Leading Coefficient

- the graph extends from quadrant 2 to quadrant 4 (similar to the graph of $y = -x$)

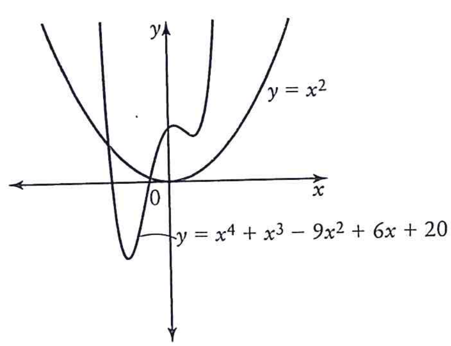


- Odd-degree polynomials have at least one x-intercept, up to a maximum of n x-intercepts, where n is the degree of the function.
- The domain of all odd-degree polynomials is $\{x \in \mathbb{R}\}$ and the range is $\{y \in \mathbb{R}\}$. Odd-degree functions have no maximum point and no minimum point.
- Odd-degree polynomials may have point symmetry.

Key Features of Graphs of Polynomial Functions With Even Degree

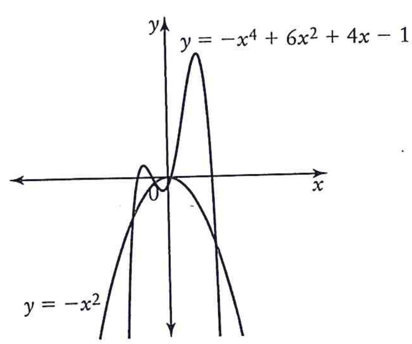
Positive Leading Coefficient

- the graph extends from quadrant 2 to quadrant 1 (similar to the graph of $y = x^2$)
- the range is $\{y \in \mathbb{R}, y \geq a\}$, where a is the minimum value of the function
- an even-degree polynomial with a positive leading coefficient will have at least one minimum point



Negative Leading Coefficient

- the graph extends from quadrant 3 to quadrant 4 (similar to the graph of $y = -x^2$)
- the range is $\{y \in \mathbb{R}, y \leq a\}$, where a is the maximum value of the function
- an even-degree polynomial with a negative leading coefficient will have at least one maximum point



- Even-degree polynomials may have from zero to a maximum of n x-intercepts, where n is the degree of the function.
- The domain of all even-degree polynomials is $\{x \in \mathbb{R}\}$.
- Even-degree polynomials may have line symmetry.

Key Features of Graphs of Polynomial Functions

- A polynomial function of degree n, where n is a whole number greater than 1, may have at most $n - 1$ local minimum and local maximum points.
- For any polynomial function of degree n, the nth differences
  - are equal (or constant)
  - have the same sign as the leading coefficient
  - are equal to $a[n \times (n - 1) \times \ldots \times 2 \times 1]$, where a is the leading coefficient

Communicate Your Understanding

C1 Describe the similarities between

a) the lines $y = x$ and $y = -x$ and the graphs of other odd-degree polynomial functions

b) the parabolas $y = x^2$ and $y = -x^2$ and the graphs of other even-degree polynomial functions

C2 Discuss the relationship between the degree of a polynomial function and the following features of the corresponding graph:

a) the number of x-intercepts

b) the number of maximum and minimum points

c) the number of local maximum and local minimum points

C3 Sketch the graph of a quartic function that

a) has line symmetry

b) does not have line symmetry

C4 Explain why even-degree polynomials have a restricted range. What does this tell you about the number of maximum or minimum points?

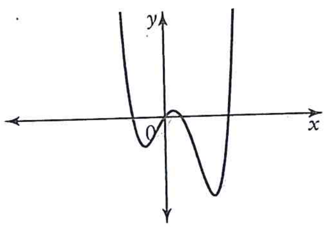
EX 1.2 (p. 26–28) #1–8

A Practise

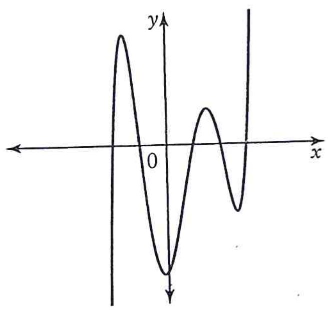
For help with questions 1 to 3, refer to Example 1.

1. Each graph represents a polynomial function of degree 3, 4, 5, or 6. Determine the least possible degree of the function corresponding to each graph. Justify your answer.

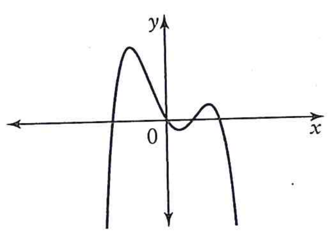
a)



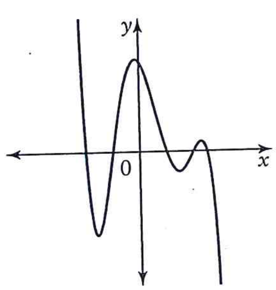
b)



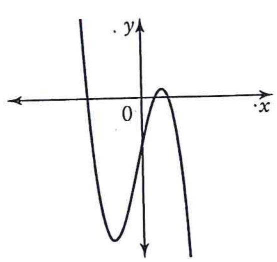
c)



d)

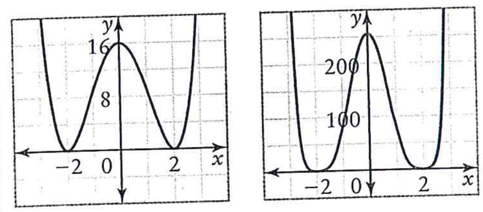


e)



CONNECTIONS

The least possible degree refers to the fact that it is possible for the graphs of two polynomial functions with either odd degree or even degree to *appear* to be similar, even though one may have a higher degree than the other. For instance, the graphs of $y = (x - 2)^2(x + 2)^2$ and $y = (x - 2)^4(x + 2)^4$ have the same shape and the same x-intercepts, -2 and 2, but one function has a double root at each of these values, while the other has a quadruple root at each of these values.



2. Refer to question 1. For each graph, do the following.

a) State the sign of the leading coefficient. Justify your answer.

b) Describe the end behaviour.

c) Identify any symmetry.

d) State the number of minimum and maximum points and local minimum and local maximum points. How are these related to the degree of the function?

3. Use the degree and the sign of the leading coefficient to

i) describe the end behaviour of each polynomial function

ii) state which finite differences will be constant

iii) determine the value of the constant finite differences

a) $f(x) = x^2 + 3x - 1$

b) $g(x) = -4x^3 + 2x^2 - x + 5$

c) $h(x) = -7x^4 + 2x^3 - 3x^2 + 4$

d) $p(x) = 0.6x^5 - 2x^4 + 8x$

e) $f(x) = 3 - x$

f) $h(x) = -x^6 + 8x^3$

For help with question 4, refer to Example 2.

4. State the degree of the polynomial function that corresponds to each constant finite difference. Determine the value of the leading coefficient for each polynomial function.

a) second differences = −8

b) fourth differences = −48

c) third differences = −12

d) fourth differences = 24

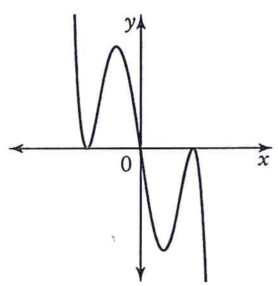
e) third differences = 36

f) fifth differences = 60

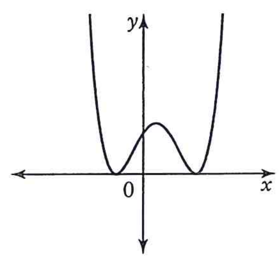
B Connect and Apply

5. Determine whether each graph represents an even-degree or an odd-degree polynomial function. Explain your reasoning.

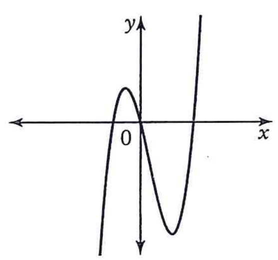
a)



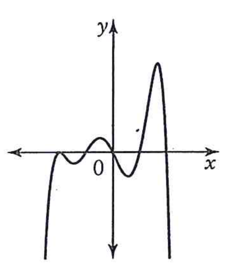
b)



c)



d)



6. Refer to question 5. For each graph, do the following.

a) State the least possible degree.

b) State the sign of the leading coefficient.

c) Describe the end behaviour of the graph.

d) Identify the type of symmetry, if it exists.

For help with question 7, refer to Example 2.

7. Each table represents a polynomial function. Use finite differences to determine the following for each polynomial function.

i) the degree

ii) the sign of the leading coefficient

iii) the value of the leading coefficient

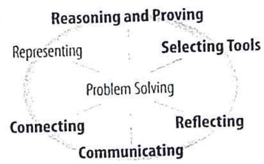
a)

x	y
−3	−45
−2	−16
−1	−3
0	0
1	−1
2	0
3	9
4	32

b)

x	y
−2	−40
−1	12
0	20
1	26
2	48
3	80
4	92
5	30

For help with questions 8 and 9, refer to Example 3.



8. A snowboard manufacturer determines that its profit, P, in thousands of dollars, can be modelled by the function $P(x) = x + 0.001\,25x^4 - 3$, where x represents the number, in hundreds, of snowboards sold.

a) What type of function is $P(x)$?

b) Without calculating, determine which finite differences are constant for this polynomial function. What is the value of the constant finite differences? Explain how you know.

c) Describe the end behaviour of this function, assuming that there are no restrictions on the domain.

d) State the restrictions on the domain in this situation.

e) What do the x-intercepts of the graph represent for this situation?

f) What is the profit from the sale of 3000 snowboards?

9. Use Technology The table shows the displacement, s, in metres, of an inner tube moving along a waterslide after time, t, in seconds.

t (s)	s (m)
0	10
1	34
2	42
3	46
4	58
5	90
6	154
7	262

a) Use finite differences to

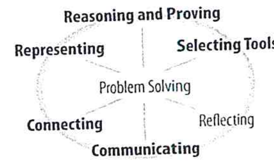
i) identify the type of polynomial function that models s

ii) determine the value of the leading coefficient

b) Graph the data in the table using a graphing calculator. Use the regression feature of the graphing calculator to determine an equation for the function that models this situation.

10. a) Sketch graphs of $y = \sin x$ and $y = \cos x$.

b) Compare the graph of a periodic function to the graph of a polynomial function. Describe any similarities and differences. Refer to the end behaviour, local maximum and local minimum points, and maximum and minimum points.



11. The volume, V, in cubic centimetres, of a collection of open-topped boxes can be modelled by $V(x) = 4x^3 - 220x^2 + 2800x$, where x is the height of each box, in centimetres.

a) Graph $V(x)$. State the restrictions.

b) Fully factor $V(x)$. State the relationship between the factored form of the equation and the graph.

c) State the value of the constant finite differences for this function.

12. A medical researcher establishes that a patient's reaction time, r, in minutes, to a dose of a particular drug is $r(d) = -0.7d^3 + d^2$, where d is the amount of the drug, in millilitres, that is absorbed into the patient's blood.

a) What type of function is $r(d)$?

b) Without calculating the finite differences, state which finite differences are constant for this function. How do you know? What is the value of the constant differences?

c) Describe the end behaviour of this function if no restrictions are considered.

d) State the restrictions for this situation.

13. By analysing the impact of growing economic conditions, a demographer establishes that the predicted population, P, of a town t years from now can be modelled by the function $p(t) = 6t^4 - 5t^3 + 200t + 12\,000$.

a) Describe the key features of the graph represented by this function if no restrictions are considered.

b) What is the value of the constant finite differences?

c) What is the current population of the town?

d) What will the population of the town be 10 years from now?

e) When will the population of the town be approximately 175 000?