Ex 1.4 (p. 49 - 52) #1 – 10, #12

KEY CONCEPTS

- The graph of a polynomial function of the form $y = a[k(x - d)]^n + c$ can be sketched by applying transformations to the graph of $y = x^n$, where $n \in \mathbb{N}$. The transformations represented by a and k must be applied before the transformations represented by c and d.
- The parameters a, k, d, and c in polynomial functions of the form $y = a[k(x - d)]^n + c$, where n is a non-negative integer, correspond to the following transformations:
  - a corresponds to a vertical stretch or compression and, if $a < 0$, a reflection in the x-axis
  - k corresponds to a horizontal stretch or compression and, if $k < 0$, a reflection in the y-axis
  - c corresponds to a vertical translation up or down
  - d corresponds to a horizontal translation to the left or right

Communicate Your Understanding

C1 **a)** Which parameters cause the graph of a power function to become wider or narrower?

b) Describe what values of the parameters in part a) make a graph

i) wider **ii)** narrower

C2 Which parameters do not change the shape of a power function? Provide an example.

C3 Which parameters can cause the graph of a power function to be reflected? Describe the type of reflections.

C4 **a)** Describe the order in which the transformations should be applied to obtain an accurate graph.

b) What sequences of transformations produce the same result?

A Practise

For help with question 1, refer to Example 1.

1. a) The graph of $y = x^4$ is transformed to obtain the graph of $y = 4[3(x + 2)]^4 - 6$. State the parameters and describe the corresponding transformations.

b) Copy and complete the table.

$y = x^4$	$y = (3x)^4$	$y = 4(3x)^4$	$y = 4[3(x + 2)]^4 - 6$
(−2, 16)			
(−1, 1)			
(0, 0)			
(1, 1)			
(2, 16)			

c) Sketch a graph of $y = 4[3(x + 2)]^4 - 6$.

d) State the domain and range, the vertex, and the equation of the axis of symmetry.

For help with questions 2 to 4, refer to Example 2.

2. Match each function with the corresponding transformation of $y = x^n$.

a) $y = -x^n$

b) $y = (-x)^n + 2$

c) $y = -(-x)^n$

d) $y = x^n$

i) no reflection

ii) reflection in the x-axis

iii) reflection in the x-axis and the y-axis

iv) reflection in the y-axis

3. Match each function with the corresponding transformation of $y = x^n$.

a) $y = 2x^n$

b) $y = (2x)^n$

c) $y = \frac{1}{2}x^n$

d) $y = \left(\frac{1}{2}x\right)^n$

i) horizontally stretched by a factor of 2

ii) vertically compressed by a factor of $\frac{1}{2}$

iii) vertically stretched by a factor of 2

iv) horizontally compressed by a factor of $\frac{1}{2}$

4. Compare each polynomial function with the equation $y = a[k(x - d)]^n + c$. State the values of the parameters a, k, d, and c and the degree n, assuming that the base function is a power function. Describe the transformation that corresponds to each parameter.

a) $y = (3x)^3 - 1$

b) $y = 0.4(x + 2)^2$

c) $y = x^3 + 5$

d) $y = \frac{3}{4}[-(x - 4)]^3 + 1$

e) $y = 2\left(\frac{1}{3}x\right)^4 - 5$

f) $y = 8[(2x)^3 + 3]$

For help with question 5, refer to Example 3.

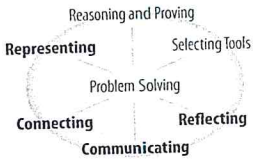
5. Match each graph with the corresponding function. Justify your choice.

i) $y = -\frac{1}{4}x^3$

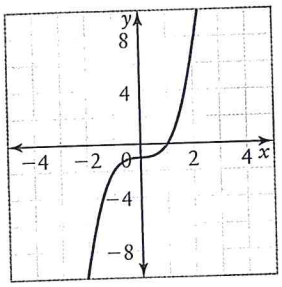
ii) $y = x^3 - 1$

iii) $y = \left(-\frac{1}{4}x\right)^5$

iv) $y = -x^4$



a)



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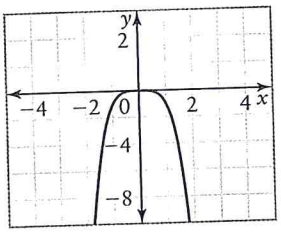

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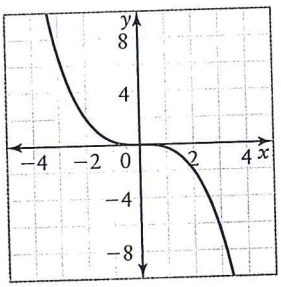

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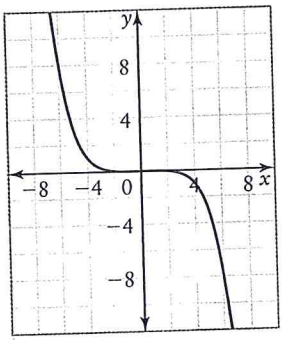


B Connect and Apply

For help with questions 6 to 8, refer to Example 2.

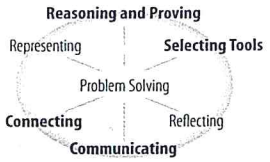
6. Describe the transformations that must be applied to the graph of each power function $f(x)$ to obtain the transformed function. Write the transformed equation.

a) $f(x) = x^2$, $y = f(x + 2) - 1$

b) $f(x) = x^3$, $y = f(x - 4) + 5$

7. a) Given a base function of $y = x^4$, list the parameters of the polynomial function

$$y = -3\left[\frac{1}{2}(x + 4)\right]^4 + 1.$$



b) Describe how each parameter in part a) transforms the graph of the function $y = x^4$.

c) Determine the domain, range, vertex, and equation of the axis of symmetry for the transformed function.

d) Describe two possible orders in which the transformations can be applied to the graph of $y = x^4$ in order to sketch the graph of

$$y = -3\left[\frac{1}{2}(x + 4)\right]^4 + 1.$$

8. Describe the transformations that must be applied to the graph of each power function, $f(x)$, to obtain the transformed function. Write the full equation of the transformed function.

a) $f(x) = x^3$, $y = -0.5f(x - 4)$

b) $f(x) = x^4$, $y = -f(4x) + 1$

c) $f(x) = x^3$, $y = 2f\left[\frac{1}{3}(x - 5)\right] - 2$

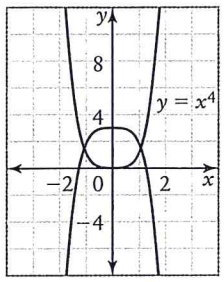
9. a) For each pair of polynomial functions in question 8, sketch the original and transformed functions on the same set of axes.

b) State the domain and range of the functions in each pair. For even functions, give the vertex and the equation of the axis of symmetry.

10. i) Transformations are applied to each power function to obtain the resulting graph. Determine an equation for the transformed function.

ii) State the domain and range. For even functions, give the vertex and the equation of the axis of symmetry.

a)



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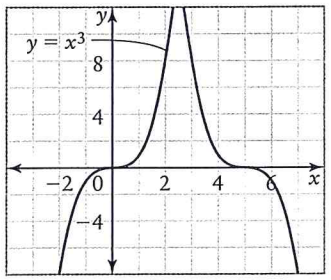

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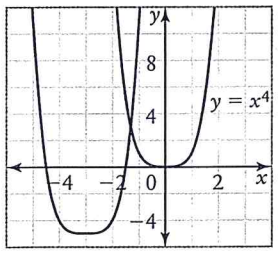
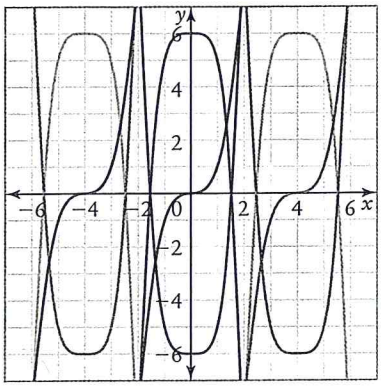


11. Chapter Problem A mechanical engineer is experimenting with new designs of fibreglass furnace filters to improve air quality. One of the patterns being considered for the new design is shown, superimposed on a grid. Determine equations for the polynomial functions used to create this pattern.



12. i) Write an equation for the function that results from the given transformations.

ii) State the domain and range. For even functions, give the vertex and the equation of the axis of symmetry.

a) The function $f(x) = x^4$ is translated 2 units to the left and 3 units up.

b) The function $f(x) = x^5$ is stretched horizontally by a factor of 5 and translated 12 units to the left.

c) The function $f(x) = x^4$ is stretched vertically by a factor of 3, reflected in the x-axis, and translated 6 units down and 1 unit to the left.

d) The function $f(x) = x^6$ is reflected in the x-axis, stretched horizontally by a factor of 5, reflected in the y-axis, and translated 3 units down and 1 unit to the right.

e) The function $f(x) = x^6$ is compressed horizontally by a factor of $\frac{4}{5}$, stretched vertically by a factor of 7, reflected in the x-axis, and translated 1 unit to the left and 9 units up.

✓Achievement Check

13. a) The graph of $y = x^4$ is transformed to obtain the graph of $y = \frac{1}{4}[-2(x - 1)]^4 + 2$. List the parameters and describe the corresponding transformations.

b) Sketch a graph of $y = \frac{1}{4}[-2(x - 1)]^4 + 2$.

C Extend and Challenge

14. a) Predict the relationship between the graph of $y = x^3 - x^2$ and the graph of $y = (x - 2)^3 - (x - 2)^2$.

b) **Use Technology** Graph each function using technology to verify the accuracy of your prediction.

c) Factor each function in part a) to determine the x-intercepts.

15. Use Technology

a) Describe the transformations that must be applied to the graph of $y = x^4 - x^3 + x^2$ to obtain the graph of $y = -3\left(\left[\frac{1}{2}(x + 4)\right]^4 - \left[\frac{1}{2}(x + 4)\right]^3 + \left[\frac{1}{2}(x + 4)\right]^2\right)$.

b) Sketch each graph using technology.

c) Factor each function in part a) to determine the x-intercepts.

16. a) The function $h(x) = 3(x - 3)(x + 2)(x - 5)$ is translated 4 units to the left and 5 units down. Write an equation for the transformed function.

b) Suppose the transformed function is then reflected in the x-axis and vertically compressed by a factor of $\frac{2}{5}$. Write an equation for the new transformed function.

17. Math Contest A farm has a sale on eggs, selling 13 eggs for the usual price of a dozen eggs. As a result, the price of eggs is reduced by 24 cents a dozen. What was the original price for a dozen eggs?

18. Math Contest Given $f_0(x) = x^2$ and $f_{n+1} = f_0(f_n(x))$, where n is any natural number

a) determine $f_1(x)$, $f_2(x)$, and $f_3(x)$

b) determine a formula for $f_n(x)$ in terms of n