 (p. 74 _ 77) # 1 — 18

### 1.1 Power Functions

**1. i)** Which functions are polynomial functions? Justify your answer.

**ii)** State the degree and the leading coefficient of each polynomial function.

**a)** $f(x) = 3x^4 - 5x + 1$

**b)** $g(x) = x(4 - x)$

**c)** $h(x) = 3x + 2x$

**d)** $m(x) = x^{-2}$

**e)** $r(x) = 5(x - 1)^3$

**2.** For each graph, do the following.

**i)** State whether the corresponding function has even degree or odd degree.

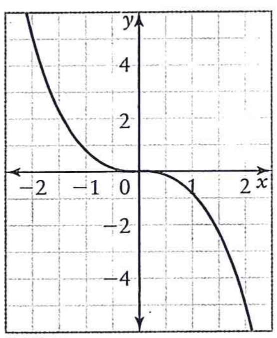
**ii)** State whether the leading coefficient is positive or negative.

**iii)** State the domain and range.

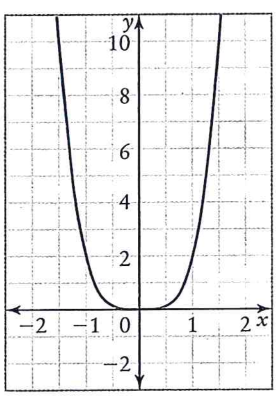
**iv)** Describe the end behaviour.

**v)** Identify the type of symmetry.

**a)**




**b)**




**3.** Set up a table as shown. Write each function in the appropriate row of column 2. Give reasons for your choices.

$y = -x^5$, $y = \frac{2}{3}x^4$, $y = 4x^3$, $y = 0.2x^6$

| End Behaviour | Function | Reasons |
|---|---|---|
| Extends from quadrant 3 to quadrant 1 | | |
| Extends from quadrant 2 to quadrant 4 | | |
| Extends from quadrant 2 to quadrant 1 | | |
| Extends from quadrant 3 to quadrant 4 | | |

### 1.2 Characteristics of Polynomial Functions

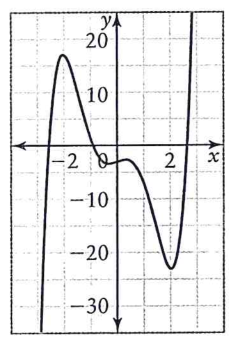
**4.** Match each graph of a polynomial function with the corresponding equation. Justify your choice.

**i)** $g(x) = 0.5x^4 - 3x^2 + 5x$

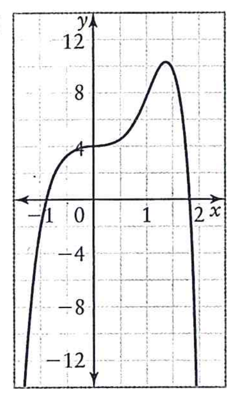
**ii)** $h(x) = x^5 - 7x^3 + 2x - 3$

**iii)** $p(x) = -x^6 + 5x^3 + 4$

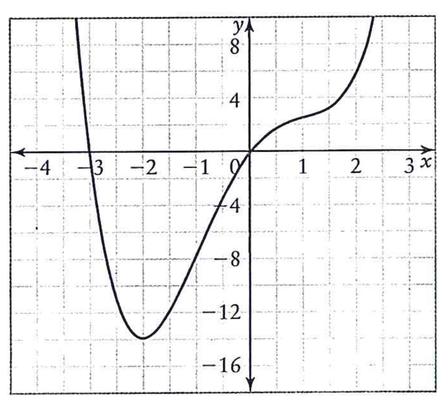
**a)**




**b)**




**c)**



5. For each polynomial function in question 4, do the following.

a) Determine which finite differences are constant.

b) Find the value of the constant finite differences.

c) Identify the type of symmetry, if it exists.

6. a) State the degree of the polynomial function that corresponds to each constant finite difference.

i) first differences = $-5$

ii) fifth differences = $-60$

iii) fourth differences = 36

iv) second differences = 18

v) third differences = 42

vi) third differences = $-18$

b) Determine the value of the leading coefficient of each polynomial function in part a).

7. Each table of values represents a polynomial function. Use finite differences to determine the following for each:

i) the degree

ii) the sign of the leading coefficient

iii) the value of the leading coefficient

a)

| $x$ | $y$ |
|---|---|
| $-3$ | 124 |
| $-2$ | 41 |
| $-1$ | 8 |
| 0 | 1 |
| 1 | $-4$ |
| 2 | $-31$ |
| 3 | $-104$ |
| 4 | $-247$ |

b)

| $x$ | $y$ |
|---|---|
| $-2$ | $-229$ |
| $-1$ | $-5$ |
| 0 | 3 |
| 1 | $-7$ |
| 2 | $-53$ |
| 3 | $-129$ |
| 4 | 35 |
| 5 | 1213 |

8. A parachutist jumps from a plane 3500 m above the ground. The height, $h$, in metres, of the parachutist above the ground $t$ seconds after the jump can be modelled by the function $h(t) = 3500 - 4.9t^2$.

a) What type of function is $h(t)$?

b) Without calculating the finite differences, determine

i) which finite differences are constant for this polynomial function

ii) the value of the constant finite differences

Explain how you found your answers.

c) Describe the end behaviour of this function assuming there are no restrictions on the domain.

d) Graph the function. State any reasonable restrictions on the domain.

e) What do the $t$-intercepts of the graph represent for this situation?

## 1.3 Equations and Graphs of Polynomial Functions

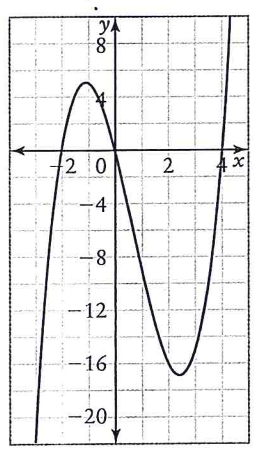
9. Use each graph of a polynomial function to determine

i) the least possible degree and the sign of the leading coefficient

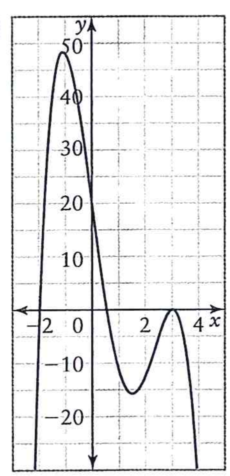
ii) the $x$-intercepts and the factors of the function

iii) the intervals where the function is positive and the intervals where it is negative

a)




b)



10. Sketch a graph of each polynomial function.

a) $y = (x + 1)(x - 3)(x + 2)$

b) $y = -x(x + 1)(x + 2)^2$

c) $y = (x - 4)^2(x + 3)^3$

11. The zeros of a quartic function are $-3$, $-1$, and 2 (order 2). Determine

a) equations for two functions that satisfy this condition

b) an equation for a function that satisfies this condition and passes through the point $(1, 4)$

12. Without graphing, determine if each polynomial function has line symmetry about the $y$-axis, point symmetry about the origin, or neither. Graph the functions to verify your answers.

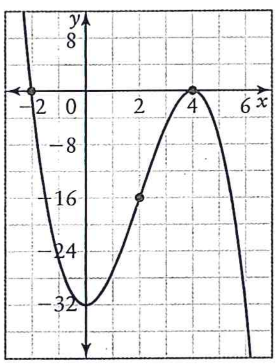
a) $f(x) = -x^5 + 7x^3 + 2x$

b) $f(x) = x^4 + 3x^2 + 1$

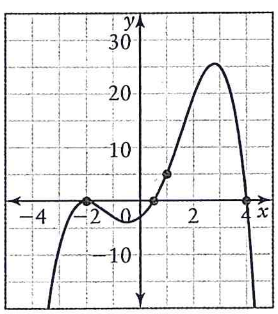
c) $f(x) = 4x^3 - 3x^2 + 8x + 1$

13. Determine an equation for the polynomial function that corresponds to each graph.

a)




b)




## 1.4 Transformations

14. i) Describe the transformations that must be applied to the graph of each power function, $f(x)$, to obtain the transformed function. Then, write the corresponding equation.

ii) State the domain and range of the transformed function. For even functions, state the vertex and the equation of the axis of symmetry.

a) $f(x) = x^3$, $y = -\frac{1}{4}f(x) - 2$

b) $f(x) = x^4$, $y = 5f\left[\frac{2}{5}(x - 3)\right] + 1$

15. i) Write an equation for the function that results from each set of transformations.

ii) State the domain and range. For even functions, state the vertex and the equation of the axis of symmetry.

a) $f(x) = x^4$ is compressed vertically by a factor of $\frac{3}{5}$, stretched horizontally by a factor of 2, reflected in the $y$-axis, and translated 1 unit up and 4 units to the left.

b) $f(x) = x^3$ is compressed horizontally by a factor of $\frac{1}{4}$, stretched vertically by a factor of 5, reflected in the $x$-axis, and translated 2 units to the left and 7 units up.

## 1.5 Slopes of Secants and Average Rate of Change

16. Which are not examples of average rates of change? Explain why.

a) The average height of the players on the basketball team is 2.1 m.

b) The temperature of water in the pool decreased by 5°C over 3 days.

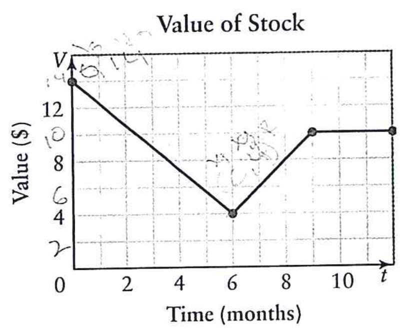
c) The snowboarder raced across the finish line at 60 km/h.

d) The class average on the last math test was 75%.

e) The value of the Canadian dollar increased from \$0.75 U.S. to \$1.01 U.S. in 8 months.

f) Approximately 30 cm of snow fell over a 5-h period.

17. The graph represents the approximate value of a stock over 1 year.




a) What was the value of the stock at the start of the year? at the end of the year?

b) What does the graph tell you about the average rate of change of the value of the stock in each interval?

i) month 0 to month 6

ii) month 6 to month 9

iii) month 9 to month 12

c) Determine the average rate of change of the value of the stock for the time periods in part b). Interpret these values for this situation.

## 1.6 Slopes of Tangents and Instantaneous Rate of Change

18. The table shows the percent of Canadian households that used the Internet for electronic banking.

| Year | Households (%) |
|---|---|
| 1999 | 8.0 |
| 2000 | 14.7 |
| 2001 | 21.6 |
| 2002 | 26.2 |
| 2003 | 30.8 |

Source: Statistics Canada, Canada at a Glance 2006, page 9, Household Internet use at home by Internet activity.

a) Determine the average rate of change, in percent, of households that used the Internet for electronic banking from 1999 to 2003.

b) Estimate the instantaneous rate of change in the percent of households that used the Internet for electronic banking in the year 2000, and also in 2002.

c) Compare the values you found in parts a) and b). Explain any similarities and differences.

CHAPTER

## PROBLEM WRAP-UP

Throughout this chapter, you have seen how polynomial functions can be used in different careers involving design. Create a design that uses polynomial functions. The design could be for items such as furniture, vehicles, games, clothing, or jewellery.

Provide a report on the design you select that includes the following:

- a description of the item the design relates to and why you selected it
- a drawing of the design using appropriate colours
- an explanation of how the design integrates the graphs of a variety of polynomial functions

Include the following information about each function you used to create the design:

- the equation, domain and range, and end behaviour
- the value of the constant finite differences
- any transformations used
- any symmetry
- any connections to average and instantaneous rate of change

You may wish to do some research on the Internet for ideas and you may want to use technology to help you create your design.