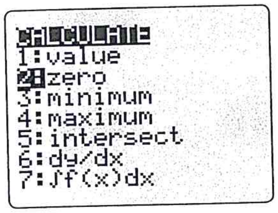
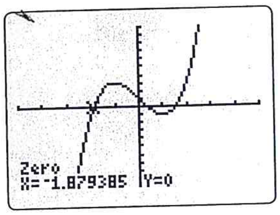
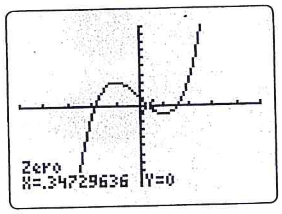
Ex2.3 (p.109 - 111) #1 - 8, and #10, 11, 14.

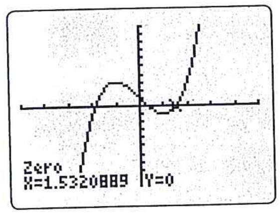
From the graph, there are three x-intercepts, one near -2, another near 0, and a third near 2.

Use the **Zero** operation.









The three roots of the equation are -1.9, 0.3, and 1.5, to one decimal place.

CONNECTIONS

Another method of solving the equation with a graphing calculator is to find the points of intersection of the graphs of the two functions $y = x^3 - 3x$ and $y = -1$.

KEY CONCEPTS

- The real roots of a polynomial equation $P(x) = 0$ correspond to the x-intercepts of the graph of the polynomial function $P(x)$.
- The x-intercepts of the graph of a polynomial function correspond to the real roots of the related polynomial equation.
- If a polynomial equation is factorable, the roots are determined by factoring the polynomial, setting its factors equal to zero, and solving each factor.
- If a polynomial equation is not factorable, the roots can be determined from the graph using technology.

Communicate Your Understanding

C1 Describe what is meant by a root, a zero, and an x-intercept. How are they related?

C2 Without solving, describe two ways to show that 2, -1, 3, and -2 are the roots of the polynomial equation $x^4 - 2x^3 - 7x^2 + 8x + 12 = 0$.

C3 A polynomial equation of degree four has exactly two distinct real roots. How many x-intercepts does the graph of the polynomial function have?

C4 Describe the different methods that can be used to factor a polynomial function.

C5 Suppose the degree of a polynomial function is n. What is the maximum number of real roots of the corresponding equation? Will the number of x-intercepts of the graph of the function be the same as the number of roots? Explain.

A Practise

For help with question 1, refer to Example 1.

1. Solve.

a) $x(x + 2)(x - 5) = 0$

b) $(x - 1)(x - 4)(x + 3) = 0$

c) $(3x + 2)(x + 9)(x - 2) = 0$

d) $(x - 7)(3x + 2)(x + 1) = 0$

e) $(4x - 1)(2x - 3)(x + 8) = 0$

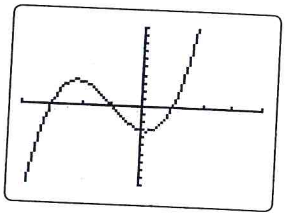
f) $(2x - 5)(2x + 5)(x - 7) = 0$

g) $(5x - 8)(x + 3)(2x - 1) = 0$

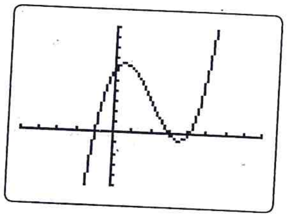
For help with question 2, refer to Example 2.

2. Use the graph to determine the roots of the corresponding polynomial equation. The roots are all integral values.

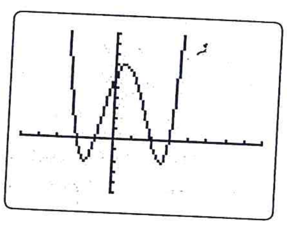
a) Window variables: $x \in [-4, 4]$, $y \in [-10, 10]$



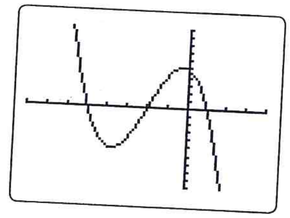
b) Window variables: $x \in [-5, 8]$, $y \in [-10, 20]$, Yscl $= 2$



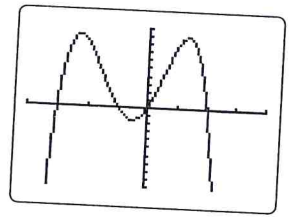
c) Window variables: $x \in [-5, 8]$, $y \in [-10, 20]$, Yscl $= 2$



d) Window variables: $x \in [-8, 4]$, $y \in [-20, 20]$, Yscl $= 2$



e) Window variables: $x \in [-4, 4]$, $y \in [-10, 10]$



For help with question 3, refer to Example 3.

3. Determine the real roots of each polynomial equation.

a) $(x^2 + 1)(x - 4) = 0$

b) $(x^2 - 1)(x^2 + 4) = 0$

c) $(3x^2 + 27)(x^2 - 16) = 0$

d) $(x^4 - 1)(x^2 - 25) = 0$

e) $(4x^2 - 9)(x^2 + 16) = 0$

f) $(x^2 + 7x + 12)(x^2 - 49) = 0$

g) $(2x^2 + 5x - 3)(4x^2 - 100) = 0$

4. Determine the x-intercepts of the graph of each polynomial function.

a) $y = x^3 - 4x^2 - 45x$

b) $f(x) = x^4 - 81x^2$

c) $P(x) = 6x^3 - 5x^2 - 4x$

d) $h(x) = x^3 + x^2 - 4x - 4$

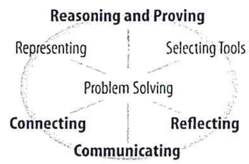
e) $g(x) = x^4 - 16$

f) $k(x) = x^4 - 2x^3 - x^2 + 2x$

g) $t(x) = x^4 - 29x^2 + 100$

B Connect and Apply

5. Is each statement true or false? If the statement is false, reword it to make it true.



a) If the graph of a quartic function has two x-intercepts, then the corresponding quartic equation has four real roots.

b) All the roots of a polynomial equation correspond to the x-intercepts of the graph of the corresponding polynomial function.

c) A polynomial equation of degree three must have at least one real root.

d) All polynomial equations can be solved algebraically.

e) All polynomial equations can be solved graphically.

6. Solve by factoring.

a) $x^3 - 4x^2 - 3x + 18 = 0$

b) $x^3 - 4x^2 - 7x + 10 = 0$

c) $x^3 - 5x^2 + 7x - 3 = 0$

d) $x^3 + x^2 - 8x - 12 = 0$

e) $x^3 - 3x^2 - 4x + 12 = 0$

f) $x^3 + 2x^2 - 7x + 4 = 0$

g) $x^3 - 3x^2 + x + 5 = 0$

7. Solve by factoring.

a) $2x^3 + 3x^2 - 5x - 6 = 0$

b) $2x^3 - 11x^2 + 12x + 9 = 0$

c) $9x^3 + 18x^2 - 4x - 8 = 0$

d) $5x^3 - 8x^2 - 27x + 18 = 0$

e) $8x^4 - 64x = 0$

f) $4x^4 - 2x^3 - 16x^2 + 8x = 0$

g) $x^4 - x^3 - 11x^2 + 9x + 18 = 0$

8. Solve by factoring.

a) $x^3 - 5x^2 + 8 = -2x$

b) $x^3 - x^2 = 4x + 6$

c) $2x^3 - 7x^2 + 10x - 5 = 0$

d) $x^4 - x^3 = 2x + 4$

e) $x^4 + 13x^2 = -36$

For help with question 9, refer to Example 4.

9. **Use Technology** Solve. Round answers to one decimal place.

a) $x^3 - 4x + 2 = 0$

b) $2x^3 + 9x^2 = x + 3$

c) $x^4 = 2$

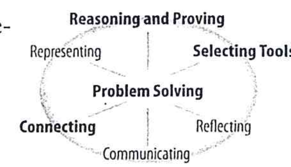
d) $3x^3 + 6 = x$

e) $x^4 = x^3 + 7$

f) $4x^3 - 3x^2 - 5x + 2 = 0$

g) $x^4 + x^2 - x + 4 = 0$

10. The width of a square-based storage tank is 3 m less than its height. The tank has a capacity of 20 m^3. If the dimensions are integer values in metres, what are they?



11. The passenger section of a train has width $2x - 7$, length $2x + 3$, and height $x - 2$, with all dimensions in metres. Solve a polynomial equation to determine the dimensions of the section of the train if the volume is 117 m^3.

12. Is it possible for a polynomial equation to have exactly one irrational root? Use an example to justify your answer.

13. Is it possible for a polynomial equation to have exactly one non-real root? Use an example to justify your answer.

14. The distance, d, in kilometres, travelled by a plane after t hours can be represented by $d(t) = -4t^3 + 40t^2 + 500t$, where $0 \le t \le 10$. How long does the plane take to fly 4088 km?

15. A steel beam is supported by two vertical walls. When a 1000-kg weight is placed on the beam, x metres from one end, the vertical deflection, d, in metres, can be calculated using the formula $d(x) = 0.0005(x^4 - 16x^3 + 512x)$. How far from the end of the beam should the weight be placed for a deflection of 0 m?