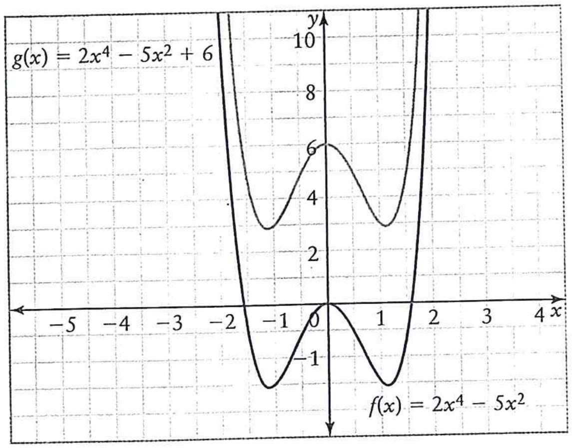


When a constant term is added to an even function, the function remains even. For example, the graph of $g(x) = 2x^4 - 5x^2 + 6$ represents a vertical translation of 6 units up of the graph of $f(x) = 2x^4 - 5x^2$. Thus, since $f(x) = 2x^4 - 5x^2$ is even and has line symmetry, the same is true for $g(x) = 2x^4 - 5x^2 + 6$.

CONNECTIONS

Recall that constant terms can be thought of as coefficients of x^0.

KEY CONCEPTS

- The graph of a polynomial function can be sketched using the x-intercepts, the degree of the function, and the sign of the leading coefficient.
- The x-intercepts of the graph of a polynomial function are the roots of the corresponding polynomial equation.
- When a polynomial function is in factored form, the zeros can be easily determined from the factors. When a factor is repeated n times, the corresponding zero has order n.
- The graph of a polynomial function changes sign only at x-intercepts that correspond to zeros of odd order. At x-intercepts that correspond to zeros of even order, the graph touches but does not cross the x-axis.
- An even function satisfies the property $f(-x) = f(x)$ for all x in its domain and is symmetric about the y-axis. An even-degree polynomial function is an even function if the exponent of each term is even.
- An odd function satisfies the property $f(-x) = -f(x)$ for all x in its domain and is rotationally symmetric about the origin. An odd-degree polynomial function is an odd function if the exponent of each term is odd.

Communicate Your Understanding

C1 Are all even-degree polynomial functions even? Are all odd-degree polynomial functions odd? Explain.

C2 Why is it useful to express a polynomial function in factored form?

C3 **a)** What is the connection between the order of the zeros of a polynomial function and the graph?

b) How can you tell from a graph if the order of a zero is 1, 2, or 3?

C4 How can symmetry be used to sketch a graph of a polynomial function?

Ex 1.3 (p. 39_40) # 1-9

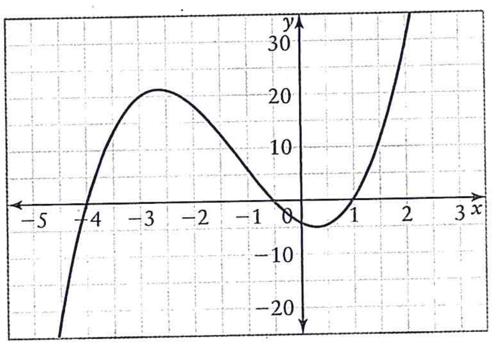
A Practise

For help with questions 1 and 2, refer to Example 1.

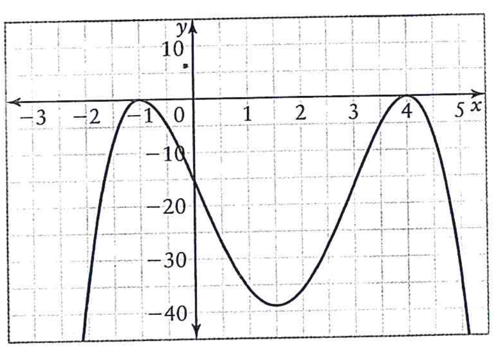
1. For each polynomial function:
   i) state the degree and the sign of the leading coefficient
   ii) describe the end behaviour of the graph of the function
   iii) determine the x-intercepts
   a) $f(x) = (x - 4)(x + 3)(2x - 1)$
   b) $g(x) = -2(x + 2)(x - 2)(1 + x)(x - 1)$
   c) $h(x) = (3x + 2)^2(x - 4)(x + 1)(2x - 3)$
   d) $p(x) = -(x + 5)^3(x - 5)^3$

2. For each graph, do the following.
   i) State the x-intercepts.
   ii) State the intervals where the function is positive and the intervals where it is negative.
   iii) Explain whether the graph might represent a polynomial function that has zeros of order 2 or of order 3.

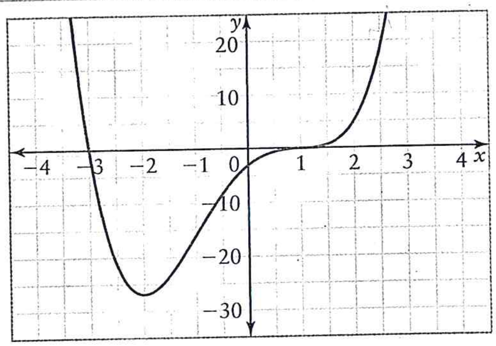
a)



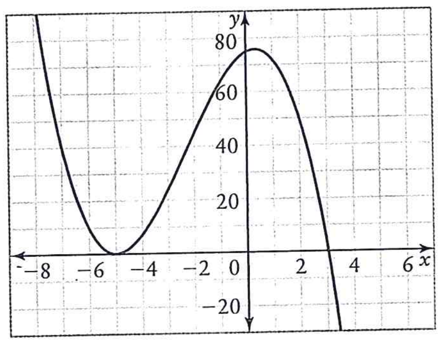
b)



c)



d)



For help with question 3, refer to Example 2.

3. a) Determine the zeros of each polynomial function. Indicate whether they are of order 1, 2, or 3.
   i) $f(x) = -2(x - 3)(x + 2)(4x - 3)$
   ii) $g(x) = (x - 1)(x + 3)(1 + x)(3x - 9)$
   iii) $h(x) = -(x + 4)^2(x - 1)^2(x + 2)(2x - 3)$
   iv) $p(x) = 3(x + 6)(x - 5)^2(3x - 2)^3$

   b) Determine algebraically if each function is even or odd.

   c) Sketch a graph of each function in part a).

For help with questions 4 and 5, refer to Example 3.

4. Determine, algebraically, whether each function in question 1 has point symmetry about the origin or line symmetry about the y-axis. State whether each function is even, odd, or neither. Give reasons for your answer.

5. i) Determine whether each function even, odd, or neither. Explain.

ii) Without graphing, determine if each polynomial function has line symmetry about the y-axis, point symmetry about the origin, or neither. Explain.

a) $y = x^4 - x^2$

b) $y = -2x^3 + 5x$

c) $y = -4x^5 + 2x^2$

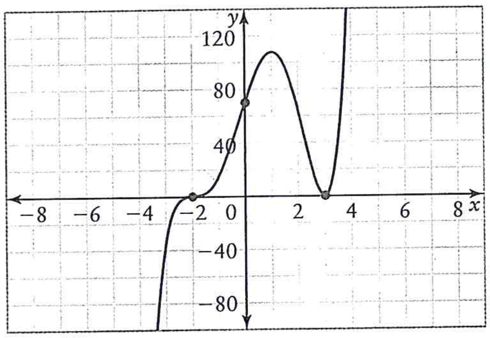
d) $y = x(2x + 1)^2(x - 4)$

e) $y = -2x^6 + x^4 + 8$

B Connect and Apply

6. Determine an equation for the polynomial function that corresponds to each graph.

a)



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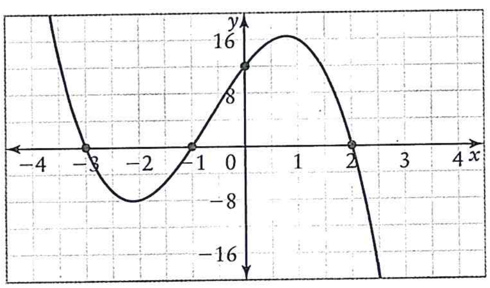
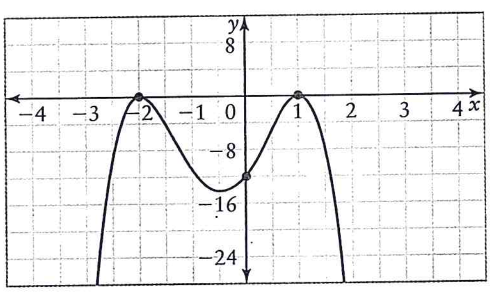
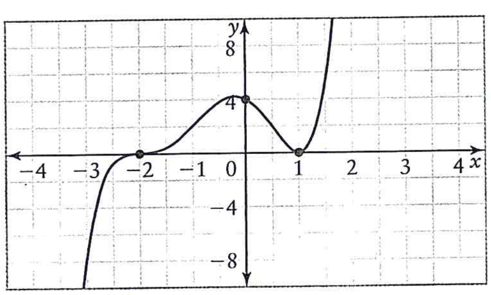


c)



d)



7. Determine an equation for each polynomial function. State whether the function is even, odd, or neither. Sketch a graph of each.

a) a cubic function with zeros -2 (order 2) and 3 and y-intercept 9

b) a quartic function with zeros -1 (order 3) and 1 and y-intercept -2

c) a quintic function with zeros -1 (order 3) and 3 (order 2) that passes through the point $(-2, 50)$

d) a quintic function with zeros -3, -2 (order 2), and 2 (order 2) that passes through the point $(1, -18)$

8. Without graphing, determine if each polynomial function has line symmetry, point symmetry, or neither. Verify your response using technology.

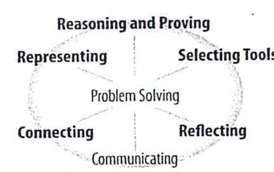
a) $f(x) = -6x^5 + 2x$

b) $g(x) = -7x^6 + 3x^4 + 6x^2$

c) $h(x) = x^3 - 3x^2 + 5x$

d) $p(x) = -5x^3 + 2x$

9. Each polynomial function has zeros at $-3, -1, 2$. Write an equation for each function. Then, sketch a graph of the function.



a) a cubic function with a positive leading coefficient

b) a quartic function that touches the x-axis at -1

c) a quartic function that extends from quadrant 3 to quadrant 4

d) a quintic function that extends from quadrant 3 to quadrant 1

10. Chapter Problem An engineer designs a rollercoaster so that a section of the ride can be modelled by the function $h(x) = -0.000\,000\,4x(x - 15)(x - 25)(x - 45)^2(x - 60)$, where x is the horizontal distance from the boarding platform, in metres; $x \in [0, 60]$; and h is the height, in metres, above or below the boarding platform.

a) What are the similarities and differences between this polynomial function and those studied in Sections 1.1 and 1.2?

b) What useful information does this form of the equation provide that can be used to sketch a graph of the path of the rollercoaster?

c) Use the information from part b) to sketch a graph of this section of the rollercoaster.

d) Estimate the maximum and the minimum height of the rollercoaster relative to the boarding platform.

11. a) Determine the zeros of $f(x) = (2x^2 - x - 1)(x^2 - 3x - 4)$.

b) Use graphing technology to verify your answer.

12. a) Determine the zeros of each polynomial function.

i) $f(x) = x^4 - 13x^2 + 36$

ii) $g(x) = 6x^5 - 7x^3 - 3x$

b) State whether each function is even, odd, or neither. Verify your answers algebraically.

c) Sketch a graph of each function.

C Extend and Challenge

13. Use Technology Consider the polynomial function $f(x) = (x - 3)(x - 1)(x + 2)^2 + c$, where c is a constant. Determine a value of c such that the graph of the function has each number of x-intercepts. Justify your answer graphically.

a) four

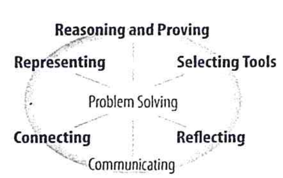
b) three

c) two

d) one

e) zero

14. a) Write equations for two even functions with four x-intercepts, two of which are $\frac{2}{3}$ and 5.



b) Determine an equation for a function with x-intercepts at $\frac{2}{3}$ and 5, passing through the point $(-1, -96)$.

c) Determine an equation for a function with x-intercepts at $\frac{2}{3}$ and 5 that is a reflection in the x-axis of the function in part b).

15. Explain algebraically why a polynomial that is an odd function is no longer an odd function when a nonzero constant is added.

16. Math Contest If the value of a continuous function $f(x)$ changes sign in an interval, there is a root of the equation $f(x) = 0$ in that interval. For example, $f(x) = x^3 - 4x - 2$ has a zero between 2 and 3. Evaluate the function at the endpoints and at the midpoint of the interval. This gives $f(2) = -2$, $f(2.5) = 3.625$, and $f(3) = 13$. The function changes sign between $x = 2$ and $x = 2.5$, so a root lies in this interval. Since $f(2.25) \doteq 0.39$, there is a root between $x = 2$ and $x = 2.25$. Continuing in this way gives increasingly better approximations of that root.

a) Determine, correct to two decimal places, the root of $x^3 - 3x + 1 = 0$ that lies between 0 and 1.

b) Calculate the greatest root of $2x^3 - 4x^2 - 3x + 1 = 0$, to three decimal places.