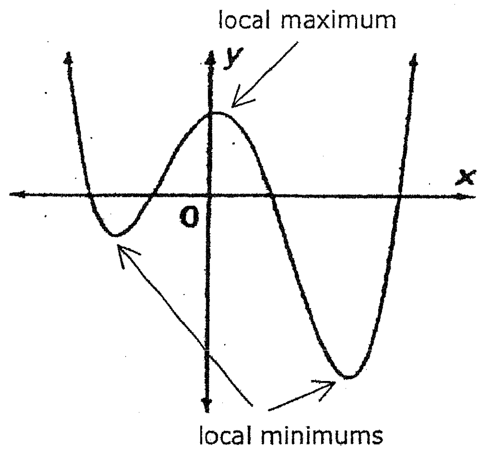
MHF 4U Date:__________

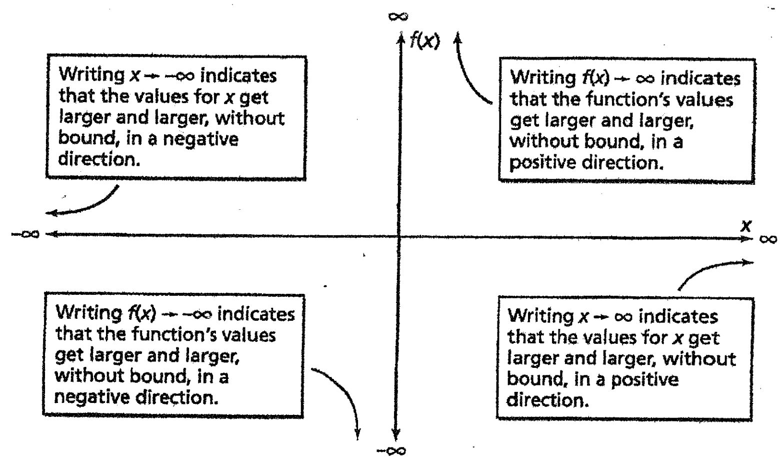
Sec. 1.2 Characteristics of Polynomial Functions

Characteristics of Polynomial Functions

A **turning point** (or critical point) is a point on a curve that is higher or lower than all nearby points. A turning point occurs where a function changes from increasing to decreasing or vice versa. When the function, $f(x)$ changes from increasing to decreasing at (x, y), then (x, y) is called a **local maximum** and $f(x) = y$ is the **local maximum value**. When the function, $f(x)$ changes from decreasing to increasing at (x, y), then (x, y) is called a **local minimum** and $f(x) = y$ is the **local minimum value.**

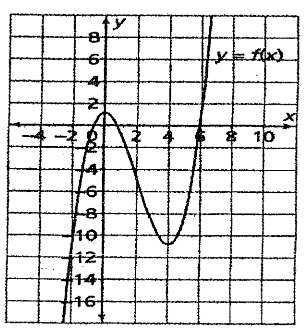


An important characteristic of any polynomial function is its **end behaviour**. End behaviour describes the values of a function, $f(x)$, as x takes on large positive, or large negative, numbers. You can describe end behaviour using the symbols ∞ and -∞, which mean positve and negative infinity, respectively. The diagram on the right shows how these symbols can be used.

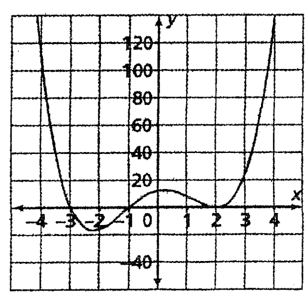


Ex 1. Describe the end behaviour of $f(x)$ as $x \rightarrow -\infty$ and as $x \rightarrow \infty$ and determine the number of local maximum and local minimum values.

a)



b)



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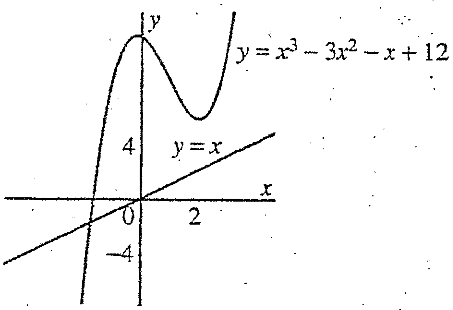
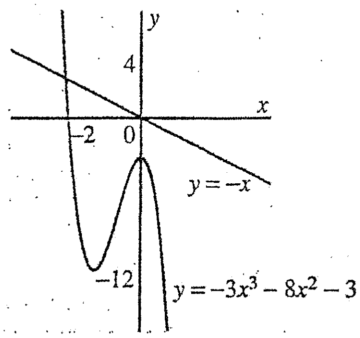
Functions with odd degree

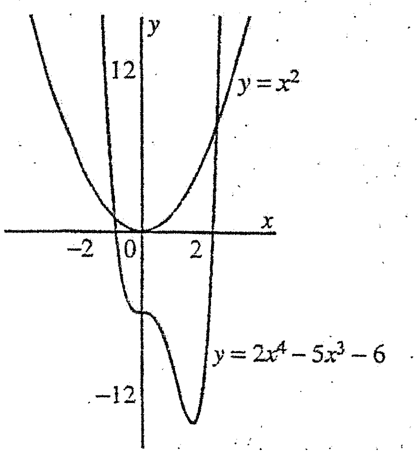


When the leading coefficient is positive, the graph extends from the 3rd quadrant to the 1st quadrant, as the graph of $y = x$ does.

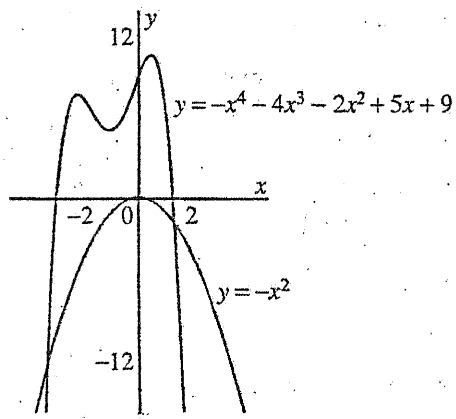


When the leading coefficient is negative, the graph extends from the 2nd quadrant to the 4th quadrant, as the graph of $y = -x$ does.

Functions with even degree



When the leading coefficient is positive, the graph extends from the 2nd quadrant to the 1st quadrant, as the graph of $y = x^2$ does.



When the leading coefficient is negative, the graph extends from the 3rd quadrant to the 4th quadrant, as the graph of $y = -x^2$ does.

Summary – Characteristics of Polynomial Functions

- The graphs of polynomial functions of degree *n* may have
  - 0 to *n* *x*-intercepts when *n* is even.
  - 1 to *n* *x*-intercepts when *n* is odd.

- The graphs of polynomial functions of degree 3 can be S-shaped curves.
- The graphs of polynomial functions of degree 4 can be W-shaped curves.

- The graphs of polynomial functions of degree n have at most n – 1 turning points.

Sec. 1.2 Characteristics of Polynomial Functions

MHF4U Numerical Properties of Polynomial Functions

1. Consider the function $y = x$
   a) What type of function is it?
   b) Complete the table of values.
   c) Calculate the first differences.
   d) In this case, the first differences were positive. How would the graph differ if the first differences were negative?

x	y	First Differences
–3		
		▶
–2		
		▶
–1		
		▶
0		
		▶
1		
		▶
2		
		▶
3		

2. Consider the function $y = x^2$
   a) What type of function is it?
   b) Complete the table of values.
   c) Calculate the first and second differences.

x	y	Differences	
		First	Second
–3			
		▶	
–2			▶
		▶	
–1			▶
		▶	
0			▶
		▶	
1			▶
		▶	
2			▶
		▶	
3			

Numerical Properties of Polynomial Functions (continued)

3. Consider the function $y = x^3$
   a) What type of function is it?
   b) Complete the table of values.
   c) Calculate the first, second, and third differences.

x	y	Differences		
		First	Second	Third
–3				
–2				
–1				
0				
1				
2				
3				

4. Consider the function $y = x^4$
   a) What type of function is it?
   b) Complete the table of values.
   c) Calculate the first, second, third, and fourth differences.

x	y	Differences			
		First	Second	Third	Fourth
–3					
–2					
–1					
0					
1					
2					
3					

5. a) Summarize the patterns you observe in Questions 1–4.

   b) Hypothesize as to whether or not your patterns hold when values for the b, c, d, and k parameters are not equal to 0 in $y = ax + k$, $y = ax^2 + bc + k$, $y = ax^3 + bx^2 + cx + k$, and $y = ax^4 + bx^3 + cx^2 + dx + k$.